New Companding Transform for PAPR Reduction in OFDM

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***Abstract*—High peak-to-average power ratio (PAPR) is a major drawback of orthogonal frequency division multiplexing (OFDM) systems. Among the various PAPR reduction techniques, companding transform appears attractive for its simplicity and effectiveness. This paper proposes a new companding algorithm. Compared with the others, the proposed algorithm offers an improved bit error rate and minimized out-of-band interference while reducing PAPR effectively. Theoretical analysis and numerical simulation are presented.**

***Index Terms*—Companding, OFDM, PAPR.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has been attracting substantial attention due to its excellent performance under severe channel condition [1]. The rapidly growing application of OFDM includes WiMAX, DVB/DAB and 4G wireless systems.

However, OFDM is not without drawbacks. One critical problem is its high peak-to-average power ratio (PAPR) [1]. High PAPR increases the complexity of analog-to-digital (A/D) and digital-to-analog (D/A) converters, and lowers the efficiency of power amplifiers. Over the past decade various PAPR reduction techniques have been proposed, such as block coding, selective mapping (SLM) and tone reservation, just to name a few [2]. Among all these techniques the simplest solution is to clip the transmitted signal when its amplitude exceeds a desired threshold. Clipping is a highly nonlinear process, however. It produces significant out-of-band interference (OBI).

A good remedy for the OBI is the so-called companding. The technique 'soft' compresses, rather than 'hard' clips, the signal peak and causes far less OBI. The method was first proposed in [3], which employed the classical μ-law transform and showed to be rather effective. Since then many different companding transforms with better performances have been published [4]-[7].

This paper proposes and evaluates a new companding algorithm. The algorithm uses the special airy function and is able to offer an improved bit error rate (BER) and minimized OBI while reducing PAPR effectively.

The paper is organized as follows. In the next section the PAPR problem in OFDM is briefly reviewed. Section III presents the new algorithm and its theoretical analysis, followed by the performance simulation in Section IV. The last section draws the conclusion.

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II. PAPR in OFDM

Let $X(0), X(1), \cdots, X(N-1)$ represent the data sequence to be transmitted in an OFDM symbol with N subcarriers. The baseband representation of the OFDM symbol is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{\frac{j2\pi nt}{N}} \qquad 0 \le t \le T, \tag{1}$$

where T is the duration of the OFDM symbol. According to the central limit theorem, when N is large, both the real and imaginary parts of $x(t)$ become Gaussian distributed, each with zero mean and a variance of $E[|x(t)|^2]/2$, and the amplitude of the OFDM symbol follows a Rayleigh distribution. Consequently it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. Practical hardware (e.g. A/D and D/A converters, power amplifiers) has finite dynamic range; therefore the peak amplitude of OFDM signal must be limited.

PAPR is mathematically defined as:

$$\text{PAPR} = 10 \log_{10} \frac{\max[|x(t)|^2]}{\frac{1}{T}\int_0^T |x(t)|^2 \, dt} \quad (\text{dB}). \tag{2}$$

It is easy to see from (2) that PAPR reduction may be achieved by decreasing the numerator $\max[|x(t)|^2]$, increasing the denominator $(1/T) \cdot \int_0^T |x(t)|^2 \, dt$, or both.

The effectiveness of a PAPR reduction technique is measured by the complementary cumulative distribution function (CCDF), which is the probability that PAPR exceeds some threshold, i.e.:

$$\text{CCDF} = \text{Probability}(\text{PAPR} > p_0), \tag{3}$$

where p_0 is the threshold.

III. New Companding Algorithm

OBI is the spectral leakage into alien channels. Quantification of the OBI caused by companding requires the knowledge of the power spectral density (PSD) of the companded signal. Unfortunately analytical expression of the PSD is in general mathematically intractable, because of the nonlinear companding transform involved. Here we take an alternative approach to estimate the OBI. Let $f(x)$ be a nonlinear companding function, and $x(t) = \sin(\omega t)$ be the input to the compander. The companded signal $y(t)$ is:

$$y(t) = f[x(t)] = f\left[\sin(\omega t)\right]. \tag{4}$$

Since $y(t)$ is a periodic function with the same period as $x(t)$, $y(t)$ can then be expanded into the following Fourier series:

$$y(t) = \sum_{k=-\infty}^{+\infty} c(k) e^{jk\omega t}, \tag{5}$$

where the coefficients $c(k)$ is calculated as:

$$c(k) = c(-k) = \frac{1}{T}\int_0^T y(t)e^{-jk\omega t}dt \qquad T = \frac{2\pi}{\omega}. \tag{6}$$

Notice that the input x in this case is a pure sinusoidal signal, any $c(k) \neq 0$ for $|k| > 1$ is the OBI produced by the nonlinear companding process. Therefore, to minimize the OBI, $c(k)$ must approach to zero fast enough as k increases. It has been shown that $c(k) \cdot k^{-(m+1)}$ tends to zero if $y(t)$ and its derivative up to the m-th order are continuous [8], or in other words, $c(k)$ converges at the rate of $k^{-(m+1)}$. Given an arbitrary number n, the n-th order derivative of $y(t)$, $d^n y/dt^n$, is a function of $d^i f(x)/dx^i$, $(i = 1, 2, \cdots, n)$, as well as $\sin(\omega t)$ and $\cos(\omega t)$, i.e.:

$$\frac{d^n y}{dt^n} = g\left(\frac{d^n f(x)}{dx^n}, \frac{d^{n-1} f(x)}{dx^{n-1}}, \cdots, \frac{df(x)}{dx}, \sin(\omega t), \cos(\omega t)\right). \tag{7}$$

$\sin(\omega t)$ and and $\cos(\omega t)$ are continuous functions, $d^n y/dt^n$ is continuous if and only if $d^i f(x)/dx^i$ $(i = 1, 2, \cdots, n)$ are continuous. Based on this observation we can conclude:

Companding introduces minimum amount of OBI if the companding function $f(x)$ is infinitely differentiable.

The functions that meet the above condition are the smooth functions.

We now propose a new companding algorithm using a smooth function, namely the airy special function. The companding function is as follows:

$$f(x) = \beta \cdot \text{sign}(x) \cdot [\text{airy}(0) - \text{airy}(\alpha \cdot |x|)], \tag{8}$$

where airy$(\cdot)$ is the airy function of the first kind. α is the parameter that controls the degree of companding (and ultimately PAPR). β is the factor adjusting the average output power of the compander to the same level as the average input power:

$$\beta = \sqrt{\frac{E\left[|x|^2\right]}{E\left[|\text{airy}(0) - \text{airy}(\alpha \cdot |x|)|^2\right]}}, \tag{9}$$

where $E[\cdot]$ denotes the expectation.

The decompanding function is the inverse of $f(x)$:

$$f^{-1}(x) = \frac{1}{\alpha} \cdot \text{sign}(x) \cdot \text{airy}^{-1}\left[\text{airy}(0) - \frac{|x|}{\beta}\right], \tag{10}$$

where the superscript -1 represents the inverse operation. Notice that the input to the decompander is a quantized signal with finite set of values. We can therefore numerically pre-compute $f^{-1}(x)$ and use table look-up to perform the decompanding in practice.

Next we examine the BER performance of the algorithm. Let $y(t)$ denote the output signal of the compander, $w(t)$ the white Gaussian noise. The received signal can be expressed as:

$$z(t) = y(t) + w(t). \tag{11}$$

The decompanded signal $\tilde{x}(t)$ simply is:

$$\tilde{x}(t) = f^{-1}[z(t)] = f^{-1}[y(t) + w(t)]. \tag{12}$$

Notice that the signal-to-noise ratio (SNR) in a typical additive white Gaussian noise (AWGN) channel is much greater than 1. Using the first order Taylor series expansion, (12) can be approximated as follows:

$$\tilde{x}(t) \approx x(t) + \frac{df^{-1}(u)}{du}|_{u=y(t)} \cdot w(t). \tag{13}$$

Equation (13) shows that if $y(t)$ falls into the range of the decompanding function $f^{-1}(u)$ where $df^{-1}(u)/du|_{u=y(t)} < 1$, the noise $w(t)$ is suppressed, and if $y(t)$ is out of the range, $df^{-1}(u)/du|_{u=y(t)} > 1$ and the noise is enhanced. Therefore, if the parameter α in (8) is properly chosen such that more $y(t)$ is within the noise-suppression range of $f^{-1}(u)$, it is possible to achieve better overall BER performance. It is worth to mention though that BER and PAPR affect each other adversely and therefore there is a tradeoff to make.

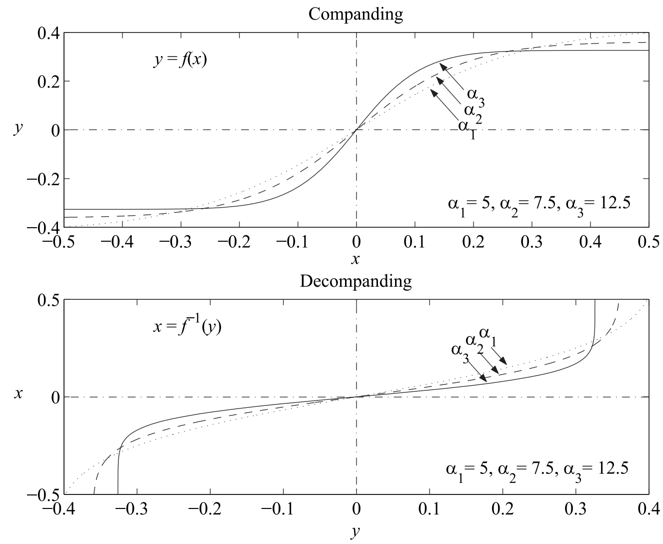

Fig. 1. Companding and decompanding profile.

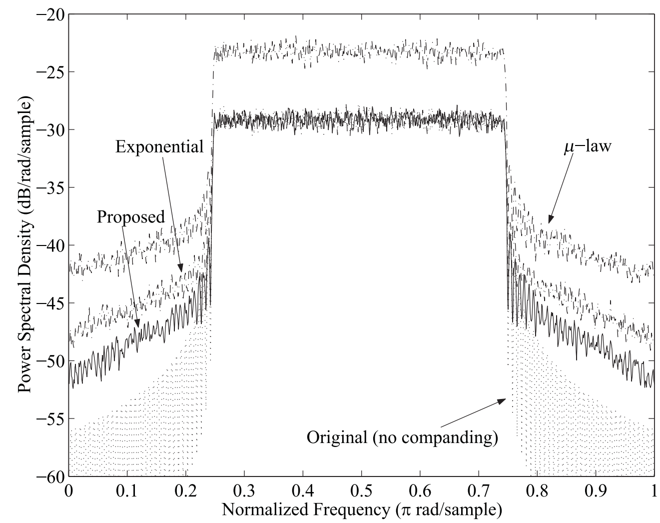

Fig. 2. Power spectral density of original and companded signals (compander input power = 3dBm, α = 30).

IV. Performance Simulation

The OFDM system used in the simulation consists of 64 QPSK-modulated data points. The size of the FFT/IFFT is 256, meaning a $4\times$ oversampling. Given the compander input

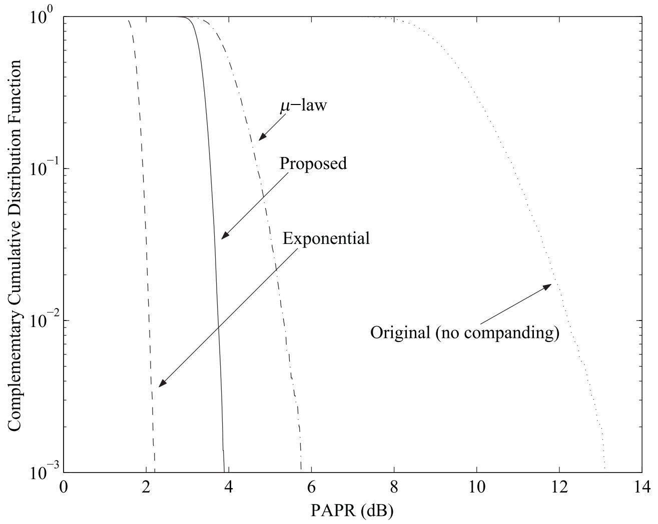

Fig. 3. Complementary cumulative distribution function of original and companded signals (compander input power = 3dBm, α = 30).

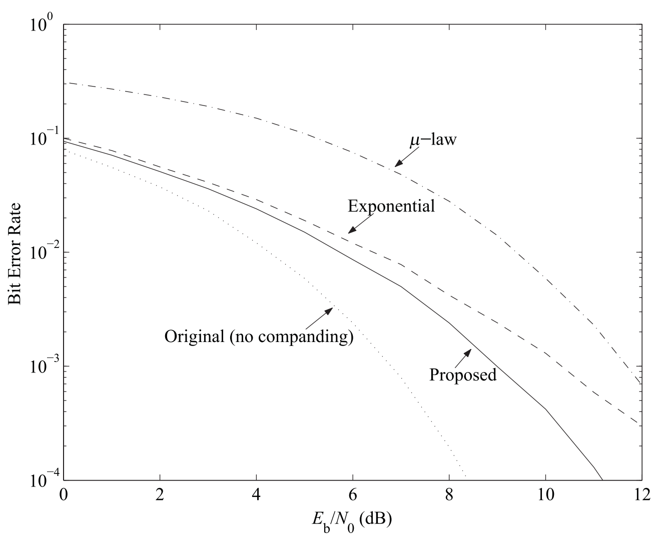

Fig. 4. Bit error rate vs. SNR for original and companded signals in AWGN channel (compander input power = 3dBm, α = 30).

power of 3dBm, the parameter α in the companding function is chosen to be 30. Consequently about 19.6 percent of $y(t)$ is within the noise-suppression range of the decompanding function. Two other popular companding algorithms, namely the μ-law companding [3] and the exponential companding [5], are also included in the simulation for the purpose of performance comparison.

The simulated PSD of the companded signals is illustrated in Fig. 2. The proposed algorithm produces OBI almost 3dB lower than the exponential algorithm, 10dB lower than the μ-law. The result is in line with our expectation. The μ-law function has a singularity in its second order derivative at $x = 0$ and therefore is expected to have the strongest OBI.

Fig. 3 depicts the CCDF of the three companding schemes. The new algorithm is roughly 1.5dB inferior to the exponential, but surpasses the μ-law by 2dB.

The BER vs. SNR is plotted in Fig. 4. Our algorithm outperforms the other two. To reach a BER of 10^{-3}, for example, the required SNR are 8.9dB, 10.4dB and 11.7dB respectively for the proposed, the exponential and the μ-law companding schemes, implying a 1.5dB and 2.8dB improvement with the new algorithm. The amount of improvement increases as SNR becomes higher.

One more observation from the simulation is: unlike the exponential companding whose performance is found almost unchanged under different degrees of companding, the new algorithm is flexible in adjusting its specifications simply by changing the value of α in the companding function.

V. Conclusion

In this paper, we have proposed a new companding algorithm. Both theoretical analysis and computer simulation show that the algorithm offers improved performance in terms of BER and OBI while reducing PAPR effectively.

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