# Wavelets For Improving Spectral Efficiency In A Digital Communication System

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## Abstract

*In this paper, two types of wavelet based schemes to improve the Spectral Efficiency (SE) of a digital communication system are presented. First one is wavelet based pulse shaping and the other is Wavelet based digital modulation called Wavelet Shift Keying (WSK). In pulse shaping scheme, orthonormal wavelets and their translates are used as base band shaping pulses .To improve spectral efficiency and coding gain, dyadic expansions and their translates are used for signaling. Since wavelets have zero average value, they can be transmitted using Single Side-Band (SSB) transmission. By comparing with Raised-Cosine (RC) signaling, this wavelet approach offers more data rate at the same bandwidth. RC system offers only 0.83 b/S/Hz for low pass transmission where as transmission using the first two dyadics offers 1.12 b/S/Hz. More over, it is shown that wavelets obey Nyquist pulse shaping conditions. Using a dyadic expansion to support a channel code there is a coding gain over the RC system, with essentially no bandwidth penalty. In Modulation scheme, the user data stream is transformed into sequence of scaled mother wavelets to indicate which version of the mother wavelet is transmitted. This modulation offers more Spectral Efficiency as number of users increases keeping the power efficiency as constant.*

***Key words -*** *Pulse shaping, Wavelets, Dyadics, Modulation, Power Spectral Density, Spectral Efficiency, Probability of error.*

## 1. Introduction

Wavelets have recently become very popular in many different scientific fields, including signal processing. A wavelet is a signal or waveform having desirable characteristics such as localization in time and frequency, and orthogonality across scale and translation. Wavelets appear to be promising waveforms due to its inherent orthonormality property [1][2].

Motivation for the use of wavelets for pulse shaping stems from the fact that Nyquist pulse-shaping criterion is satisfied by wavelets and scaling functions and also the development of Bi-orthogonal RC wavelets [3]. In fact, it turns out that the band limited RC waveforms are scaling functions of a special class of Meyer wavelets [4]. For WSK, the motivation came from the compact support of the orthonormal wavelets when compared to the pulsed sinusoid as in conventional keying.

This paper is organized as follows. Section 2 discusses the proposed system description for wavelet based pulse shaping. Section 3 gives performance comparison of the proposed signaling system with RC signaling and among different wavelet systems. Section 4 explains the proposed system for M-ary WSK. The SE of WSK is compared with conventional schemes such as BPSK and CPM in section 5 and section 6 concludes the paper.

## 2. Proposed System 1: Pulse Shaping

A block diagram of the proposed communication system is given in Figure 1. Showing its SE can recognize the importance of the above-proposed system. A SSB transmission for digital data is taken into consideration for further bandwidth efficiency.

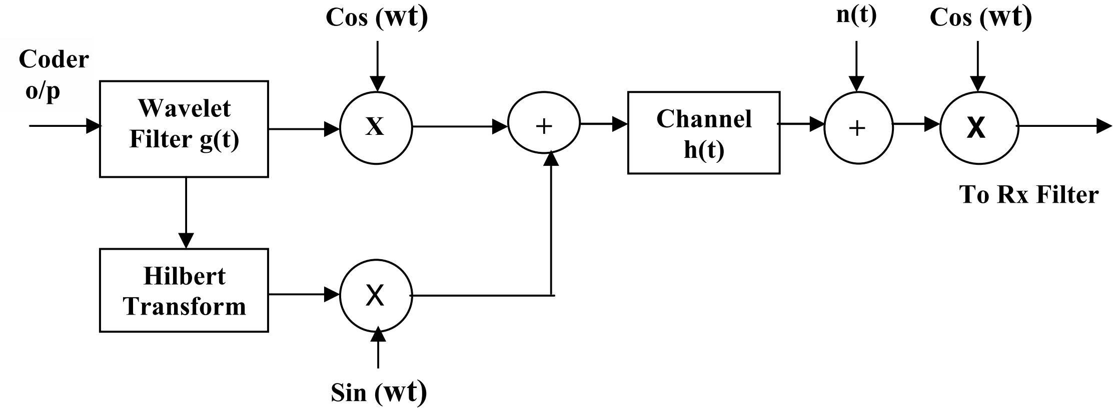


Figure 1. Proposed pass band digital communication system

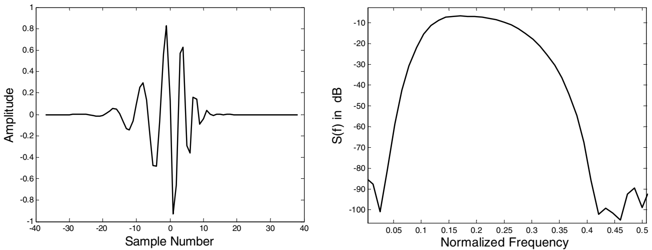
In digital communication transmission of information takes place by transmitting a sequence of ones and zeros in the form of bipolar electrical pulses. In order to transmit the sequence $a_n$, n=0,1…where $a_n \in \{\pm 1\}$, a mother wavelet as the shaping pulse is used in the modulation. To transmit a second sequence, $b_n$ , n=0,1…where $b_n \in \{\pm 1\}$, in the same bandwidth a first dyadic expansion of the mother wavelet as the shaping pulse is used.
Adding the two signals together yields

$$x_1(t) + x_2(t) = \sum_n a_n h_{0n} + \sum_m b_m h_{1m} \quad (1)$$

where $h_{0n}$ and $h_{1m}$ are the mother wavelet and its first dyadic respectively. Here note that $< h_{0n}(t), h_{1m}(t) >= 0$, i.e., the two signals are orthogonal. By continuing this process, adding dyadic expansions to transmit N data streams in a given bandwidth, yields

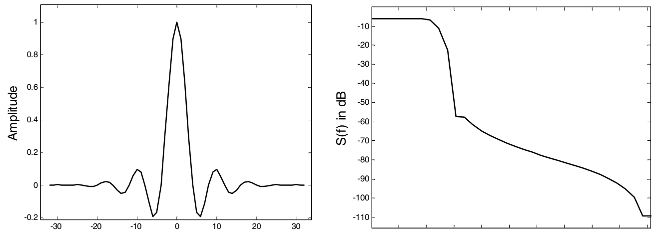
$$x(t) = \sum_{m=0}^{N} \sum_{n=0}^{N} a_{mn} . h_{mn}(t) \quad (2)$$

To recover the data after transmission, matched filter receiver was used. The wavelet considered for signaling is Daubechies of order 10 [5] and is given in Figure 2a. It was chosen because of its narrow bandwidth [6] [7]. The PSD of the mother wavelet is given in Figure 2b. The measure of bandwidth here will be the signal with 70 dB down from its maximum value. For comparison, a RC pulse with roll-off factor ($\alpha$) 0.22 is simulated and shown in Figure 3a. The PSD of this pulse is given in Figure 3b. The system is verified by sending 35 bits of data randomly using 3 wavelet pulses i.e. the signaling was carried using mother wavelet and two more dyadic expansions. The composite signal for 3 random bit streams is shown in Figure 4.



a b

Figure 2. (a) Mother wavelet (b) its PSD



a b

Figure 3. (a) RC pulse (b) PSD of RC pulse

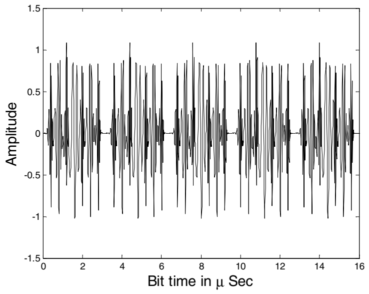


Figure 4. Composite signal

## 3. Performance Comparison

By comparing only with the mother wavelet signaling, it appears that the RC system still outperform the wavelet system. However, by using the first dyadic expansion of the mother wavelet, an additional 0.5-bit/symbol period in the same bandwidth can be transmitted. In terms of SE, the wavelet system with the mother and first dyadic pulses can now transmit 1.5 bits in 1.42/T Hz, versus 1 bit in 1.2/T Hz by the RC system. The energy per bit remains the same in the proposed system since each wavelet is an orthonormal function [8]. By adding more dyadic expansions, the system can reach an efficiency of 2 bits in 1.42/T Hz, while retaining the same energy per bit. If 2 bits per symbol are transmitted using RC pulse, it is required to increase the energy per bit by a factor of 2.5 [9]. Comparisons of SE of wavelet system versus the RC system are given in Table 1.

Since wavelets are actually band pass, the pass band bandwidths, referred to the sampling rates actually are 0.295, 0.34 and 0.358 for the mother wavelet, mother wavelet plus the first dyadic expansion and the mother wavelet plus first two dyadic expansions respectively. These are indicated as wavelet 1, wavelet 1.5 and wavelet 1.75 in the Table1.Table 2 dictates the performance comparison among different wavelet systems.

**Table 1. Comparison of SE of RC with Daubechies system**

| System | Spectral Efficiency (Base band) b/S/Hz | Spectral Efficiency (Pass band) b/S/Hz |
|---|---|---|
| RC | 0.83 | Not Applicable |
| Wavelet 1 | 0.7 | 0.85 |
| Wavelet 1.5 | 1.01 | 1.10 |
| Wavelet 1.75 | 1.12 | 1.22 |

**Table 2. Comparison among different Wavelet Systems**

| Wavelet System (Using three dyadics) | Spectral Efficiency (Base band) b/S/Hz | Spectral Efficiency (Pass band) b/S/Hz |
|---|---|---|
| Daubechies | 1.12 | 1.22 |
| Symlets | 1.05 | 1.14 |
| Coiflets | 0.95 | 1.04 |

## 3. Proposed System 2: Wavelet Shift Keying

Digital signals need to be modulated before they can be transmitted through an analogue communication channel [10]. Several coherent and non-coherent modulation waveforms have been devised for transmitting binary information over pass band channels. Common signaling schemes for transmitting digital information over band pass channels are based on a perpetual carrier wave. Wavelets have been adopted with success in many practical situations to replace sinusoidal waveforms. It seems to be somewhat expected a replacement of a classical oscillator by a waveform generator that produces a basic mother wavelet. This kind of modulation approach named Wavelet Shift Keying (WSK) and this section demonstrate the scheme.

The modulator block diagram WSK is shown in Figure 5. In the basic schemes, input binary string is converted into a stream of scale values that control the keying. The mother wavelet is locally generated and every scaled version is derived in a scaling layer. The notation X1, X2... XM specifies the scaling factor.

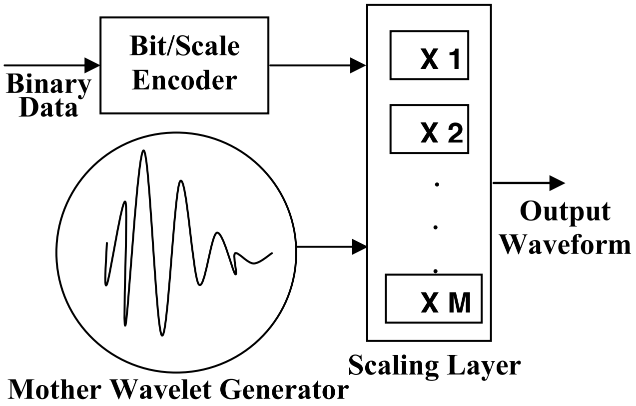


**Figure 5. M-ary WSK modulator Schematic diagram**

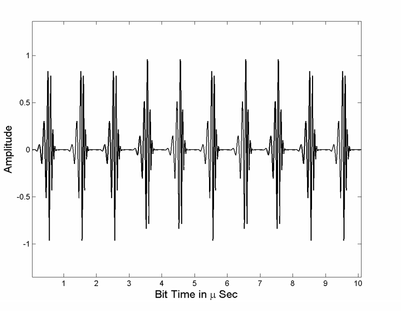


**Figure 6. Part of binary BWSK bit stream**

The transmitted WSK signal is given by

$$S_{WSK}(t) = 2^{m/2}\ \Psi(2^{m} t - n) \qquad (3)$$

where $m \in \{0,1,2...M-1\}$.

The scaled versions $\Psi(2^{m}t)$ are shorter than $\Psi(t)$ so that the transmitted signal is essentially non-overlapped wavelets. A part of (corresponding to the sequence 1 1 1 0 0 1 0 0 1 1) wavelet modulated waveform for a random 1Kbps stream is shown in Figure 6. In order to utilize WSK as a practical signaling strategy, several key properties of the waveform namely bandwidth requirements, bit error rate, AWGN performance and synchronization aspects must be quantified [11] [12]. In this work the SE and AWGN performance of the proposed scheme are explored and is compared with the conventional schemes. Other above-mentioned aspects are the areas of future research.

## 4. Performance Comparison

For comparison purposes, a 1Kbps antipodal random bit stream is modulated using BPSK, CPM and Binary WSK. The PSD of the BPSK signal before and after transmitter filtering (RC with α =. 3**)** is shown in Figure 7. The higher levels of side lobes in Figure 7a are due to the discontinuities in the phases of the carrier, when there is a transition between the bits. The PSD of the RC filtered BPSK signal shown in Figure 7b dictates a reduced side lobe signal, which can be transmitted with less interference to the neighboring spectrum occupier, which is a conventional one. Figure8 shows the PSD of the same stream Continuous Phase Modulated (with 1RC). Here the side lobes of the modulated signal itself very low due to the natural phase continuities of the CPM signal. Figure9 shows the PSD for the BWSK scheme. The wavelet used for pulse shaping scheme is used here as a keying waveform. As seen from Figure 9 WSK offers a 38 dB side lobe reduction in the absence of filtering. Also the side lobes are very far from the frequency of transmission when compared to BPSK and CPM, which has large number of reduced side lobes. Even though WSK offers a very good side lobe reduction the bandwidth required for transmission seems to be large compared with BPSK and CPM.

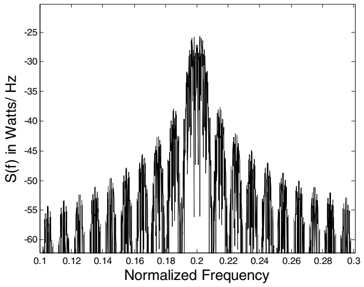


**Figure 7 (a) PSD of BPSK without filtering**

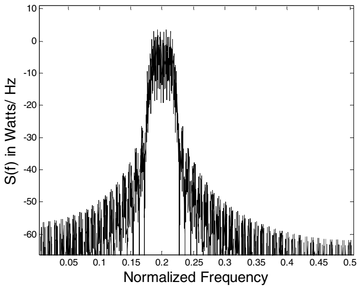


**Figure 7 (b) PSD of BPSK with RC filtering**

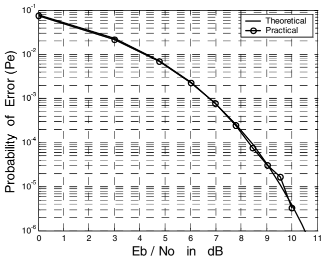


**Figure 10. Performance of BWSK**

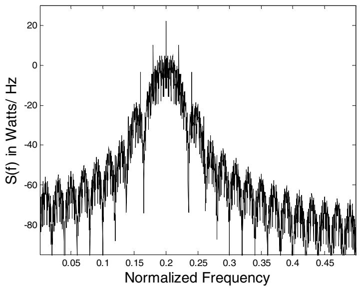


**Figure 8. PSD of CPM signal**

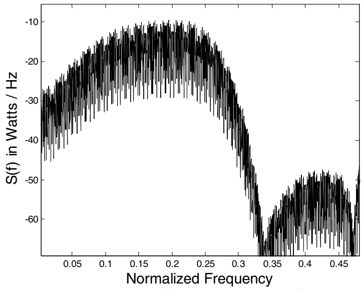


**Figure 9. PSD of BWSK signal**

The SE of the WSK seems to be discouraging at first. But, by exploiting the orthogonality property of the wavelet, the other user information can be transmitted in the same bandwidth by using the scaled mother wavelets. This considerably will increase the spectral efficiency of the wavelet system. Also, instead of using DSB-SC transmissions we can use SSB transmissions for the pass band since wavelets have zero average value [12][13]. These arguments are verified by sending more user bits by means of scaled versions of the mother wavelet through the same bandwidth along with the other users. The SE result for three users is 1.12 b/s/Hz when compared with 0.83 b/s/Hz for standard BPSK signaling with one user. In the case of BPSK if we increase the number of users the signaling set has to be increased with the resultant increase in the power of each user. But in the case of wavelets since they are orthonormal the power efficiency remains same for each bit and hence we can achieve the same power efficiency as that of BPSK still increasing the number of user transmission (SE).

Figure 10 shows the performance of BWSK in the presence of AWGN along with the performance for the coherent detection of BPSK. BWSK offers the same performance as BPSK but with more SE as the number of user increases.

## 6. Conclusion

Communication system that uses wavelets as the orthonormal basis for pulse shaping and modulation are derived. The pulse-shaping scheme can be used in a traditional PAM system to increase the spectral efficiency. The wavelet used in this paper is spectrally compact to most other wavelets found in the literature. The energy per bit remains the same when the data streams are overlapped, since each wavelet is an orthonormal function. In Conventional systems overlapping of data causes the energy per bit must be increased. Hence the proposed system is an efficient one in conserving bandwidth and power, the two important communication resources. This wavelet system can be used to increase the data rate while maintaining performance, or to increase the performance while maintaining a given data rate by accommodating a channel code using one dyadics without any bandwidth expansion. The WSK modulators proposed here can be used in the place of conventional modulators to increase the spectral efficiency. This modulator shows a very good spectral efficiency even without transmit filtering. When compared to the BPSK and CPM this WSK offers better spectral efficiency as number of users increased. The AWGN performance of the system coincides with that of BPSK. In order to utilize WSK as a practical signaling strategy, other key properties of the waveform such as performance in a fading channels and synchronization aspects must be quantified. These are the future areas to be explored.

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