A Piecewise Linear Companding Transform for PAPR Reduction of OFDM Signals With Companding Distortion Mitigation

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***Abstract*—Companding is a well-known technique for the peak-to-average power ratio (PAPR) reduction of orthogonal frequency division multiplexing (OFDM) signals. However, as companding transform is an extra operation after the modulation of OFDM signals, companding schemes reduce PAPR at the expense of increasing the bit error rate (BER). In this paper, a new piecewise linear companding scheme is proposed aiming at mitigating companding distortion. In the design of the companding transform, we study the theoretical characterization of companding distortion. It demonstrates that companding larger signals with smaller amplitude increments are more favorable in reducing companding distortion. Based on the analysis results, a new piecewise linear companding transform is proposed by clipping the signals with amplitudes over a given companded peak amplitude for peak power reduction, and linearly transforming the signals with amplitudes close to the given companded peak amplitude for power compensation. With the careful design of the companded peak amplitude and the linear transform scale, the proposed transform can achieve enhanced BER and power spectral density performance, while reducing PAPR effectively.**

***Index Terms*—OFDM, PAPR, companding transform, companding distortion.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is one of the most popular technologies in high speed wireless communication systems since the past few decades. However, despite the advantages, the inherent drawback of large envelope fluctuations of OFDM signals may cause serious performance degradation with nonlinear high power amplifier (HPA) at the transmitter.

Peak-to-average power ratio (PAPR) is widely used to characterize envelope fluctuations of OFDM signals by relating peak and mean power. Many PAPR reduction techniques have been proposed [1], [2], such as selective mapping (SLM) [3], partial transmit sequence (PTS) [4], tone reservation (TR) [5], tone injection (TI) [5], active constellation extension (ACE) [6], clipping [7] and companding [8]–[15]. Among these techniques, companding techniques have gained great attention due to their flexibility and low complexity.

In [8], Wang first proposed the $\mu -$ law companding scheme based on speech processing. However, the $\mu-$law companding scheme reduces PAPR at the expense of an increase in the average signal power. Later, another important nonlinear companding scheme namely exponential companding (EC) was developed in [10], which can obtain better PAPR reduction by transforming the distribution of OFDM signals while maintaining the average signal power constant. Recently, [12] proposes a new nonlinear companding scheme by transforming the Gaussian distributed signal into a distribution form with a linear piecewise function. Though, the nonlinear companding schemes can reduce PAPR effectively, the computational complexity of nonlinear companding is fairly high. In [13], a low-complexity linear companding transform (LCT) was introduced to reduce peak power by linearly transforming the small and large signal amplitudes with different scales. However, the average signal power cannot be kept at the same level for the input and output of LCT. Besides, as LCT does not have one-to-one mapping, additional side information was needed in the decompanding operation. To maintain the average signal power constant and to obtain a one-to-one mapping, the two-piecewise companding (TPWC) scheme investigated in [14] transforms small amplitudes with a scale and large amplitudes with both a scale and a shift. It is apparent that companding transform is an extra operation after the modulation of OFDM signals, thus companding schemes reduce PAPR at the expense of generating companding distortion. Hence, it is important for the design of companding transform aiming at minimizing the impact of companding distortion on the bit error rate (BER) performance.

In this paper, a new piecewise linear companding scheme is proposed aiming at mitigating companding distortion. In the design of the companding transform, we first investigate the theoretical characterization of the effects of companding distortion on BER. It is manifested that BER performance can be effectively improved by reducing companding distortion. Then, with a theoretical analysis of the expressions for companding distortion, we demonstrate that besides avoiding unnecessary compression in the reduction of peak power,

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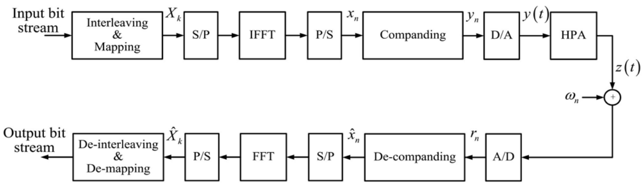


Fig. 1. Block diagram of an OFDM system with companding transform.

companding distortion can be effectively mitigated by expanding larger signals with smaller amplitude increments. Based on the analysis results, we design a new piecewise linear companding transform. In the proposed companding transform, the signals with amplitudes over a given companded peak amplitude are clipped for peak power reduction, and the signals with amplitudes close to the given companded peak amplitude are linearly scaled for power compensation. With the careful design of the companded peak amplitude and the linear transform scale, the proposed transform can effectively reduce companding distortion. Simulation results verify that the proposed companding transform can achieve enhanced BER and power spectral density (PSD) performance, while reducing PAPR effectively.

The remainder of this paper is organized as follows. Section II presents a typical OFDM system model. Section III deduces the design criteria for companding transform based on the analysis of the impact of companding distortion on SNR at the receiver. And then, the formula of the proposed companding transform is derived in Section IV. Simulation results are given in Section V. Finally, Section VI draws the conclusion for this work.

II. System Model

Fig. 1 shows the block diagram of an OFDM system with companding transform. The discrete-time transmitted OFDM signal is given by

$$x_n = \frac{1}{\sqrt{NL}} \sum_{k=0}^{NL-1} X_k e^{j2\pi \frac{kn}{NL}}, \quad 0 \le n < NL, \tag{1}$$

where $\mathbf{X} = \left[X_0, X_1, \ldots, X_{\frac{N}{2}-1}, \underbrace{0, \ldots, 0}_{N(L-1)}, X_{\frac{N}{2}}, \ldots, X_N \right]$ is the input signal vector with each data symbol modulated by QPSK or QAM. N is the number of subcarriers and L is the oversampling factor. Based on the central limit theory, x_n can be approximated as a complex Gaussian process when N is large enough. Consequently, the amplitude of x_n has a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x) = \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}} \qquad x \ge 0, \tag{2}$$

where σ_x^2 is the variance of x_n. $\sigma_x^2 = E\left[|X_k|^2\right]$, where $|\cdot|$ denotes modulus and $E[\cdot]$ is the mathematical expectation.

The PAPR of the transmitted signal can be expressed as

$$\text{PAPR} = 10 \log_{10} \frac{\max\left(|x_n|^2\right)}{E\left(|x_n|^2\right)}. \tag{3}$$

III. General Design Criteria for Companding Transform

Companding transform is an extra operation after the modulation of OFDM signals which generates companding distortion. Hence, how to reduce the impact of companding distortion on the BER performance is the key in designing companding transform. In this section, general design criteria for companding transform to reduce companding distortion are derived based on the theoretical analysis of the BER performance in terms of companding distortion.

Considering companding transform, we can regard the companded signal y_n as the original OFDM signal x_n plus an additive companding distortion signal c_n. Then, y_n can be expressed as

$$y_n = x_n + c_n. \tag{4}$$

where c_n has the same phase as x_n. Based on (4), the power of y_n can be calculated as

$$\sigma_y^2 = \mathrm{E}\left(y_n^* y_n\right) = \sigma_x^2 + 2\mathrm{E}\left(c_n^* x_n\right) + \sigma_c^2, \tag{5}$$

where $\sigma_c^2 = E\left(c_n c_n^*\right)$ is the power of c_n. As the average signal power is maintained constant in the companding operation, we get

$$\sigma_c^2 = -2\mathrm{E}\left(c_n^* x_n\right). \tag{6}$$

On the other hand, based on Bussgang Theorem [16], companded signal can also be divided into an attenuated signal part and an uncorrelated distortion part d_n. Therefore, y_n can be written as

$$y_n = \alpha x_n + d_n, \tag{7}$$

where α is the attenuating factor. α can be calculated as

$$\alpha = \frac{E\left(y_n^* x_n\right)}{E\left(|x_n|^2\right)} = 1 + \frac{\mathrm{E}\left(c_n^* x_n\right)}{\sigma_x^2}. \tag{8}$$

By Substituting (6) into (8), we have

$$\alpha = 1 - \frac{\sigma_c^2}{2\sigma_x^2}. \tag{9}$$

After the companded signal y_n passing the AWGN channel, the received signal r_n can be expressed as

$$r_n = y_n + \omega_n = \alpha x_n + d_n + \omega_n, \tag{10}$$

where ω_n is the additive white Gaussian variable, and σ_ω^2 denotes the variance of ω_n. With the decompanding operation, the recovered signal can be obtained as

$$\hat{x}_n = \frac{r_n - d_n}{\alpha} = \frac{y_n + \omega_n - d_n}{\alpha} = x_n + \frac{\omega_n}{\alpha}. \tag{11}$$

Then, the SNR at the receiver is

$$SNR = \frac{|\alpha|^2 \sigma_x^2}{\sigma_\omega^2} = \left(1 - \frac{\sigma_c^2}{2\sigma_x^2}\right) \frac{\sigma_x^2}{\sigma_\omega^2}. \tag{12}$$

It is obvious that the BER performance can be effectively improved by reducing the companding distortion σ_c^2. Since companding transform comprises of compressing and

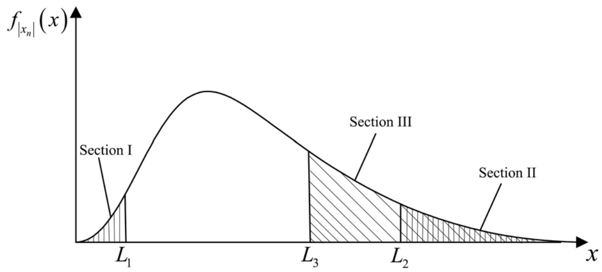


Fig. 2. PDF of the amplitude of OFDM signals.

expanding operations in the maintenance of a constant average signal power, the additive companding distortion signal c_n can be classified into two corresponding parts. Consequently, companding distortion σ_c^2 can be calculated as

$$\sigma_c^2 = \int_{\Gamma^+} |c_n|^2 f_{|x_n|}(x)\, dx + \int_{\Gamma^-} |c_n|^2 f_{|x_n|}(x)\, dx, \tag{13}$$

where Γ^+ is the sample index set corresponding to the expansion part, and Γ^- is the sample index set corresponding to the compression part. Therefore, companding distortion can be reduced by carefully designing the compressing and expanding operations, respectively.

In the design of the compressing part of companding transform, the intuitive way to mitigate companding distortion is to avoid unnecessary compressing operation. As a result, when the original signal x_n is companded with a given peak amplitude, samples whose amplitudes are below the peak amplitude should not be compressed anymore.

As for the expanding part, the design criteria are derived based on the study of the correlation between the power increase and companding distortion, for expanding is conducted to compensate the power reduction caused by compression. First, we consider a sample with amplitude x. Then, the power increase for this sample with amplitude increment Δx is

$$\begin{aligned}\Delta P_{x+\Delta x} &= P_{x+\Delta x} - P_x = (x+\Delta x)^2 - x^2 \\ &= 2x\Delta x + \Delta x^2.\end{aligned} \tag{14}$$

It is apparent that the power increase for the single sample is determined not only by the amplitude increment, but also by the sample amplitude. Moreover, as the amplitude of the OFDM signal is a random process with Rayleigh distribution, the power increase is also determined by the probability distribution. Because the probability distribution of a sample is closely related to the sample amplitude, the impacts of the probability distribution and the amplitude of the sample on power increase are considered in a unified way. To clarify the correlation of sample amplitude and amplitude increment with power increase, three sections representing different extreme situations shown in Fig. 2 are considered. To simplify the theoretical analysis, some assumptions are made in the derivation.

When considering the influence of sample amplitude on power increase, small signals in section I and large signals in section II are considered. In this situation, the probabilities p of the samples in situation I and II are assumed to be the same. Besides, the amplitude increments Δx_1 and Δx_2 in section I and II are also assumed to be equal, and we set

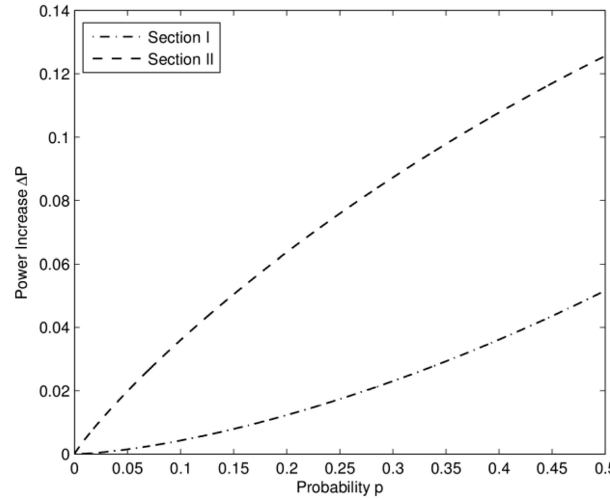


Fig. 3. Power increase of Sections I and II with the same amplitude increase, and $\Delta x_1 = 0.1$.

$\Delta x_1 = \Delta x_2$. Then, power increase for section I is obtained by

$$\Delta P_I = \int_0^{L_1} 2x\Delta x_1 f_{|x_n|}(x)dx + \int_0^{L_1} \Delta x_1^2 f_{|x_n|}(x)dx. \tag{15}$$

Power increase for section II is

$$\Delta P_{II} = \int_{L_2}^{\infty} 2x\Delta x_2 f_{|x_n|}(x)dx + \int_{L_2}^{\infty} \Delta x_2^2 f_{|x_n|}(x)dx. \tag{16}$$

Parameters L_1 and L_2 are determined by the probabilities of the samples in section I and II. In order to have a comparison on power increase between section I and II, we plot the power increase of section I and II in Fig. 3.

From Fig. 3 we can see that with the same amplitude increment, the power increase for larger amplitude is greater than that for smaller amplitude. Thus, to generate the same power increase, smaller sample index set Γ^+ or smaller amplitude increment Δx are needed for larger signals, which results in a smaller companding distortion according to (13). Therefore, companding larger signals are more favorable in the design of companding transform to reduce companding distortion.

When we evaluate how amplitude increment affects power increase, larger signals in section II and section III are considered. In the evaluation, we assume the amplitude increments Δx_2 and Δx_3 in section II and III are different, and $\Delta x_3 = \Delta x_2 / M$, where M is a positive real number. Then, power increase for section II is

$$\Delta P_{II} = \int_{L_2}^{\infty} 2x\Delta x_2 f_{|x_n|}(x)dx + \int_{L_2}^{\infty} \Delta x_2^2 f_{|x_n|}(x)dx. \tag{17}$$

Power increase for section III is

$$\Delta P_{III} = \int_{L_3}^{\infty} 2x\Delta x_3 f_{|x_n|}(x)dx + \int_{L_3}^{\infty} \Delta x_3^2 f_{|x_n|}(x)dx. \tag{18}$$

With the assumption that $\Delta P_{II} = \Delta P_{III}$, $\Delta x_2 = 0.2$ and the probability of the samples in section II equals to 0.2, we plot the curves of L_3 and σ_c^2 versus M in Fig. 4. From Fig. 4(a) we can see that L_3 decreases with the increase of M, which results in a larger sample index set Γ^+. Since amplitude increments impact companding distortion quadratically, the decrease of Δx_3 caused by the increase of M has more

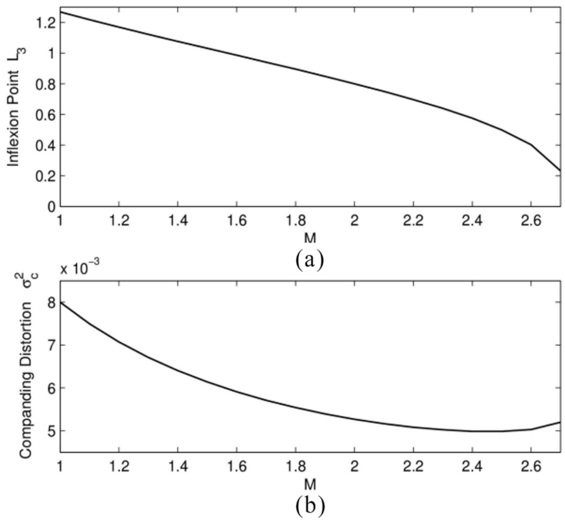


Fig. 4. Curves of L_3 and σ_c^2 in Section III versus M.

dramatic influence on σ_c^2 compared with the enlargement of the set Γ^+. Moreover, It is worth noting that when M is large enough which means Δx_3 is small enough, the enlargement of the set Γ^+ has more contribution on σ_c^2 than the decrease of Δx_3 does. Therefore, companding distortion σ_c^2 decreases with the increase of M until M arrives at certain value, which is confirmed by Fig. 4(b). Consequently, before arriving at the breakthrough point, expanding larger signals with smaller amplitude increment is more preferable in reducing companding distortion.

Based on the above analysis results, we get the general design criteria for companding transform to reduce the companding distortion which are companding transform should avoid unnecessary compression and expand larger signals with smaller amplitude increments.

IV. New Linear Companding Scheme

Based on the above design criteria for companding transform, a new piecewise linear companding scheme is proposed in this section. Then, with a theoretical analysis presented, transform parameters are carefully designed.

A. Proposed Companding Scheme

When the original signal x_n is companded with a given peak amplitude A_c, the proposed companding scheme shown in Fig. 5 clips the signals with amplitudes over A_c for peak power reduction, and linearly transforms the signals with amplitudes close to A_c for power compensation. Then, the companding function of the proposed companding scheme is

$$h(x) = \begin{cases} x & |x| \le A_i \\ kx + (1-k)A_c & A_i < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases}, \quad (19)$$

where $sgn(x)$ is the sign function.

Consequently, the decompanding function at the receiver is

$$h^{-1}(x) = \begin{cases} x & |x| \le A_i \\ (x - (1-k)A_c)\,/k & (1-k)A_c < |x| \le A_c \\ sgn(x)A_c & |x| > A_c \end{cases}, \quad (20)$$

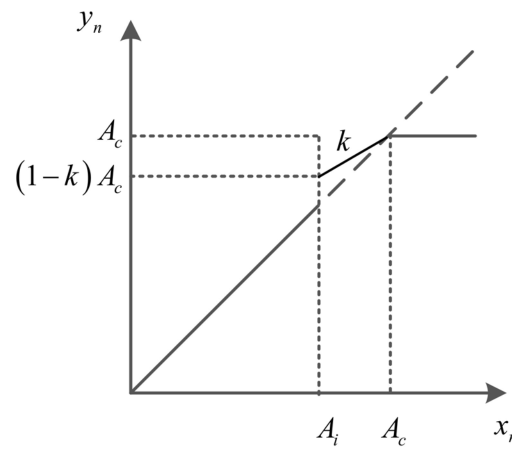


Fig. 5. Proposed linear companding transform.

It is obvious that the proposed companding transform is specified by parameters A_c, A_i and k. A_c is the peak amplitude of the companded signals. As the average signal power is maintained constant, then according to the definition of PAPR in (3), the PAPR value of the proposed scheme that can be achieved theoretically is determined by A_c. With a preset theoretical PAPR value, A_c can be determined as $A_c = \sigma_x 10^{\mathrm{PAPR_{preset}}/20}$. With determined A_c, parameters A_i and k can be obtained by solving

$$\int_{A_i}^{A_c} (kx + (1-k)A_c)^2 f_{|x_n|}(x)dx + \int_{A_c}^{\infty} A_c^2 f(x)dx = \int_{A_i}^{\infty} x^2 f_{|x_n|}(x)dx. \quad (21)$$

With appropriate manipulation, (21) can be simplified into a quadratic equation about k. The details of the manipulation of (21) are shown in Appendix. With the premise of keeping the average signal power constant, k has to be a positive real number smaller than 1. Besides, to limit the peak amplitude of the expanded signals not larger than A_c, k should not be a negative real number. Therefore, k is confined to the interval [0, 1).

B. Companding Transform Parameter Selection Criterion

Aiming at minimizing companding distortion, the selection criterion for the parameters of the proposed companding transform is derived in the sequel.

The companding distortion of the proposed companding transform can be calculated according to (13) is

$$\begin{aligned} \sigma_c^2 &= \int_0^{+\infty} |y_n - x_n|^2 f_{|x_n|}(x)dx \\ &= (1-k)^2 \left((A_c - A_i)^2 e^{-\frac{A_i^2}{\sigma_x^2}} - \sqrt{\pi}\sigma_x A_c \left(erf\left(\frac{A_c}{\sigma_x}\right) \right.\right. \\ &\quad \left.\left. - erf\left(\frac{A_i}{\sigma_x}\right)\right) + \sigma_x^2 \left(e^{-\frac{A_i^2}{\sigma_x^2}} - e^{-\frac{A_c^2}{\sigma_x^2}} \right) \right) \\ &\quad - \sqrt{\pi}\sigma_x A_c erfc\left(\frac{A_c}{\sigma_x}\right) + \sigma_x^2 e^{-\frac{A_c^2}{\sigma_x^2}} \end{aligned} \quad (22)$$

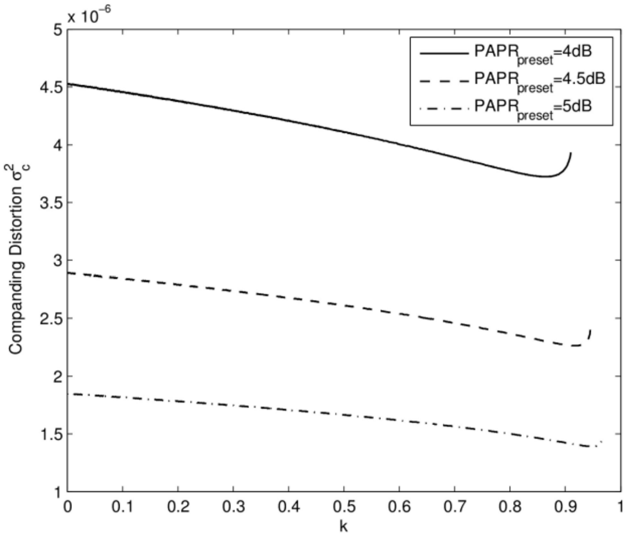


Fig. 6. Theoretical results of companding distortion of the proposed companding transform with the variation of k.

It can be seen from (22) that with a determined A_c, σ_c^2 varies with k. Therefore, for each determined A_c, we formulate the problem of solving k as an optimization problem to mitigate companding distortion

$$\begin{aligned} &\arg\min_{k\in\mathbb{R}} && \sigma_c^2 \\ &\textit{subject to}: && a_2k^2 + a_1k + a_0 = 0, \\ & && k \in [0,\ 1)\,, \\ &\textit{and} && A_c = \sigma_x e^{\mathrm{PAPR\_preset}/20}. \end{aligned} \tag{23}$$

where the first constraint is the equation (26) derived in Appendix. Fig. 6 shows the contour plot of the cost function in (23). As observed, the cost function is convex. Consequently, we can find the optimal k which leads to the minimized companding distortion for each determined A_c.

V. Performance Evaluation

To verify the performance of the proposed piecewise linear companding scheme with respect to the PAPR reduction, BER and PSD performance, numerical simulation results are presented for OFDM systems. According to IEEE 802.16 WiMAX standards, the number of subcarrier $N = 256$ is adopted for the uplink. And the proposed algorithm can also be applied in WiMAX base stations with a larger number of OFDM subcarriers. The oversampling factor L is 4. 4-QAM and 16-QAM are the baseband modulation schemes adopted in the simulations. Both the AWGN and multipath fading channels are applied. The Stanford University Interim 4 (SUI-4) is adopted as the multipath fading channel model. In the simulations, we assume perfect synchronization and channel estimation at the receiver. When considering passing companded signals through HPA, the input-output characteristics of the nonlinear region are described by Solid State Power Amplifier (SSPA) model in this paper

$$|z(t)| = \frac{|y(t)|}{\left(1 + \left(\frac{|y(t)|}{A_{sat}}\right)^{2p}\right)^{\frac{1}{2p}}} \tag{24}$$

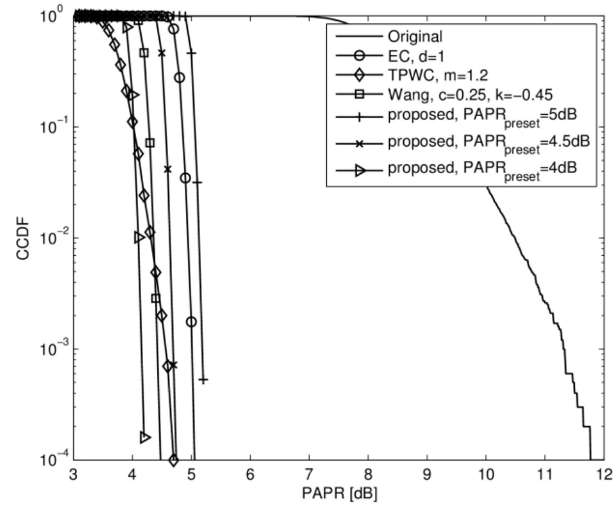


Fig. 7. CCDFs of original OFDM signal and companded signals.

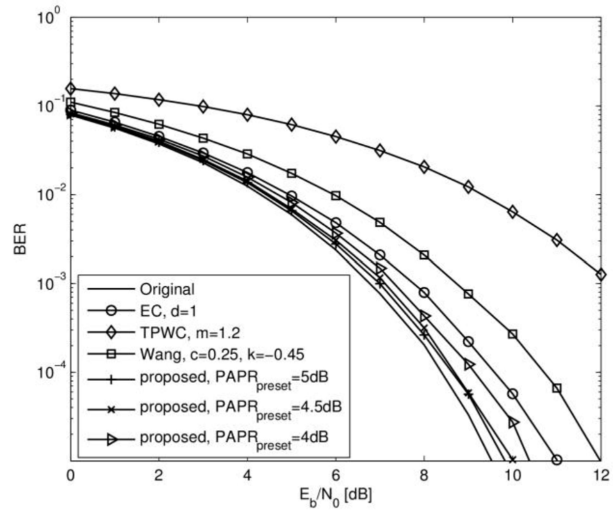


Fig. 8. BER performance of original OFDM signal and companded signals over AWGN channel with 4-QAM modulation.

where A_{sat} is the saturation level, and a typical value $p = 2$ is selected in this paper. Moreover, EC scheme ($d = 1$) [10], nonlinear companding scheme ($c = 0.25$, $k = -0.45$) [12] and TPWC scheme ($m = 1.2$) [14] are also included in the simulations for the purpose of performance comparisons. The nonlinear compaidng scheme in [12] is named as the "Wang scheme" in the performance comparisons.

Fig. 7 shows the Complementary Cumulative Distribution Function (CCDF) of PAPR of different companding schemes. As can be seen from Fig. 7, the proposed scheme can reduce PAPR effectively. Given that CCDF $= 10^{-4}$, the proposed scheme with $\mathrm{PAPR_{preset}} = 4$dB is 0.3dB, 0.5dB and 0.8dB superior over the Wang, TPWC and EC schemes, respectively.

Figs. 8 and 9 depict the BER versus E_b/N_0 curves with different companding schemes under AWGN channel with 4-QAM or 16-QAM modulation, respectively. With 4-QAM modulation, the proposed scheme achieves improved BER performance. For example, at a BER level of 10^{-3}, the proposed scheme with $\mathrm{PAPR_{preset}} = 4$dB surpasses the EC, Wang and TPWC schemes by 0.45dB, 1.4dB and 5dB, respectively.

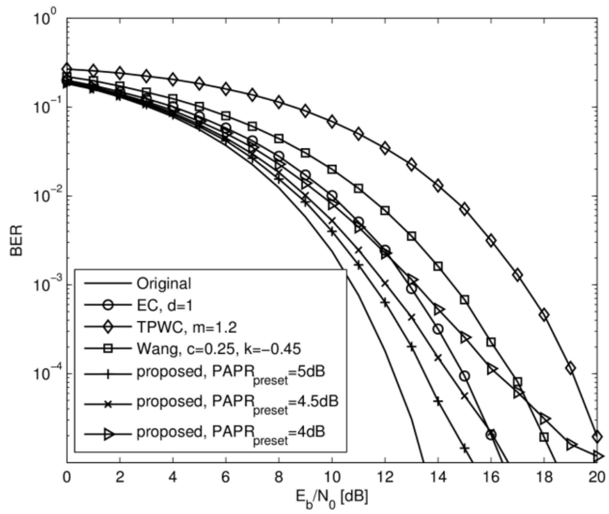


Fig. 9. BER performance of original OFDM signal and companded signals over AWGN channel with 16QAM modulation.

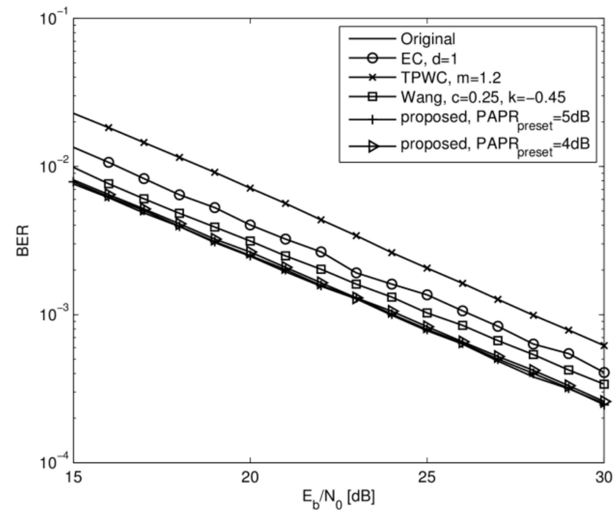


Fig. 11. BER performance of original OFDM signal and companded signals over SUI-4 channel with 4-QAM modulation.

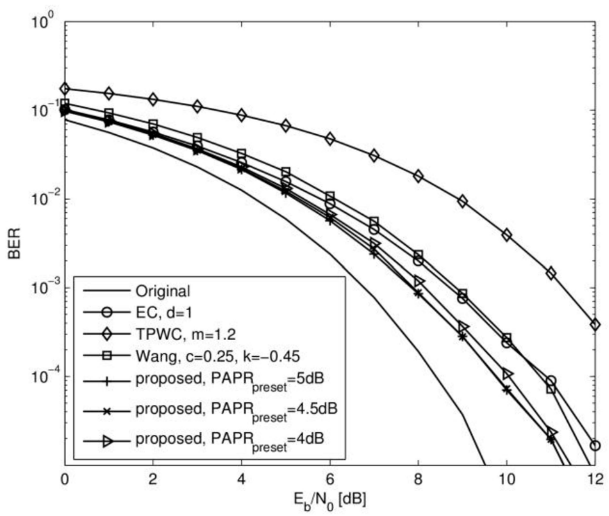


Fig. 10. BER performance of original OFDM signal and companded signals with SSPA over AWGN channel with 4-QAM modulation.

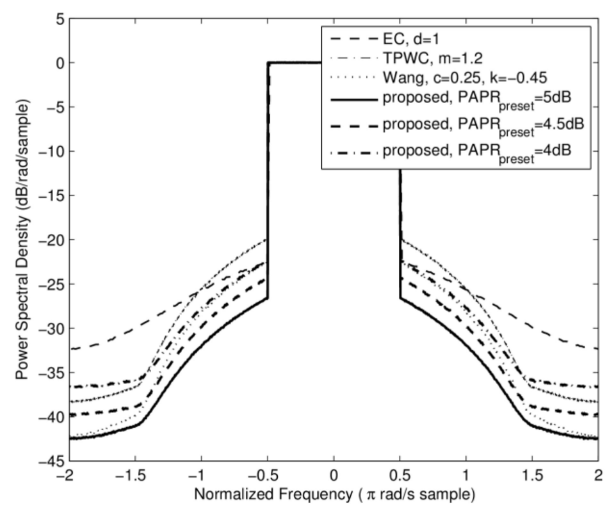


Fig. 12. PSDs of the companded signals.

With 16-QAM, the BER performance of the proposed scheme has performance floor at high SNR. The reason for this is that the output of the proposed companding function is not continuous. Without side information at the receiver, the discontinuity of companded signals will cause some ambiguity in the reconstruction of the original signal with the decompanding operation at the receiver. At a BER level of 10^{-4}, the proposed scheme with $PAPR_{preset} = 4$dB is 1.2dB inferior to the EC scheme, but is 0.6dB and 2.9dB superior over the Wang and TPWC schemes, respectively, and that with $PAPR_{preset} = 4.5$dB surpasses the EC, Wang and TPWC schemes by 0.5dB, 2.4dB and 4.7dB, respectively.

Figs. 10 and 11 present the BER performance using 4-QAM modulation with the SSPA over AWGN channel, and without the SSPA over SUI-4 channel. It can be seen that the BER performance of the proposed scheme is also robust enough with SSPA model or in the wireless Rayleigh fading channel.

Fig. 12 shows the spectral regrowth comparison among different companding schemes. To have a clear PSD comparison among these transforms, the PSDs are computed by means of periodogram. As observed, the proposed scheme with $PAPR_{preset} = 4.5$dB can achieve about 4.3dB, 1.85dB and 1.8dB out-of-band interference lower than the TPWC, EC and Wang schemes at the normalized frequency of 0.5, respectively.

Finally, a comparison of the computational efforts required by the compared schemes is made in Table I, which is shown at the top of next page. The computational complexities of the different companding schemes are measured by the required number of floating-point operations (flops). Specifically, the flop count does not include signal amplitude calculation, which is common to all these compared schemes. From Table I, we can see that as a linear companding scheme, the proposed scheme has a much smaller complexity than the nonlinear companding schemes do.

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISONS OF DIFFERENT COMPANDING SCHEMES

Companding Schemes	Flops/Sample		Number of the Companded Samples	Total Flops
	Companding	Decompanding		
EC ($d=1$)	8	7	NL	$15NL$
Wang ($c=0.25$, $k=-0.45$)	22	20	NL	$42NL$
TPWC ($m=1.2$)	1	1	NL	$2NL$
Proposed ($A_c=\sigma_x 10^{4/20}$)	1	1	$NL\int_{A_i}^{\infty} f_{\lvert x_n \rvert}(x)dx = 0.65NL$	$1.3NL$

VI. CONCLUSION

In this paper, a new piecewise linear companding scheme is proposed aiming at mitigating companding distortion to enhance the BER performance. Based on the theoretical analysis of the BER performance in terms of companding distortion, we get the general design criteria for companding transform that companding transform should avoid unnecessary compression and expand larger signals with smaller amplitude increments. Based on the design criteria, we propose a new piecewise linear companding scheme. By carefully designing the companding parameters, the proposed scheme can effectively reduce companding distortion. Simulation results verify that the proposed piecewise linear companding scheme can achieve enhanced BER and PSD performance, while reducing PAPR effectively.

APPENDIX

By substituting (2) and (19) into (21), we get

$$\int_{A_i}^{A_c} (kx + (1-k)A_c)^2 \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}} dx + \int_{A_c}^{\infty} A_c^2 \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}} dx = \int_{A_i}^{\infty} x^2 \frac{2x}{\sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}} dx. \quad (25)$$

By making appropriate simplification, a quadratic equation about k can be obtained as follows

$$a_2 k^2 + a_1 k + a_0 = 0, \quad (26)$$

Where

$$a_2 = 2A_i^2 e^{-\frac{A_i^2}{\sigma_x^2}} - \sigma_x^2 e^{-\frac{A_c^2}{\sigma_x^2}} + \sigma_x^2 e^{-\frac{A_i^2}{\sigma_x^2}} - 2A_i A_c e^{-\frac{A_i^2}{\sigma_x^2}} - \sqrt{\pi}\sigma_x A_c \left(erf\left(\frac{A_c}{\sigma_x}\right) - erf\left(\frac{A_i}{\sigma_x}\right)\right),$$

$$a_1 = 2A_i A_c e^{-\frac{A_i^2}{\sigma_x^2}} - 2A_i^2 e^{-\frac{A_i^2}{\sigma_x^2}} + \sqrt{\pi}\sigma_x A_c \left(erf\left(\frac{A_c}{\sigma_x}\right) - erf\left(\frac{A_i}{\sigma_x}\right)\right),$$

$$a_0 = -\sigma_x^2 e^{-\frac{A_i^2}{\sigma_x^2}}.$$

REFERENCES

[1] T. Jiang and Y. Wu, "An overview: Peak-to-average power ratio reduction techniques for OFDM signals," *IEEE Trans. Broadcast.*, vol. 54, no. 2, pp. 257–268, Jun. 2008.

[2] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 56–65, Apr. 2005.

[3] R. W. Bäuml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *IEEE Electron. Lett.*, vol. 32, no. 22, pp. 2056–2057, Oct. 1996.

[4] S. H. Müller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences" *IEEE Electron. Lett.*, vol. 33, no. 5, pp. 368–369, Feb. 1997.

[5] J. Tellado-Mourelo, "Peak to average power reduction for multicarrier modulation," Ph.D. thesis, Dept. Elect. Eng. Stanford Univ., Stanford, CA, USA, Sep. 1999.

[6] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," *ICASSP '03, IEEE International Conference.*, vol. 4, pp. 525–528, Apr. 2003.

[7] X. Li and L. J. Cimini, Jr., "Effects of clipping and filtering on the performance of OFDM," *Vehicular Technology Conference, 1997, IEEE 47th*, vol. 3, pp. 1634–1638, May 1997.

[8] X. Wang, T. T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using a companding technique," *IEEE Trans. Broadcast.*, vol. 45, no. 3, pp. 303–307, Sep. 1999.

[9] X. Huang, J. Lu, J. Zheng, K. B. Letaief, and J. Gu, "Companding transform for reduction in peak-to-average power of OFDM signals," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 2030–2039, Nov. 2004.

[10] T. Jiang, Y. Yang, and Y.-H. Song, "Exponential companding technique for PAPR reduction in OFDM systems," *IEEE Trans. Broadcast.*, vol. 51, no. 2, pp. 244–248, Jun. 2005.

[11] S.-S. Jeng and J.-M. Chen, "Efficient PAPR reduction in OFDM systems based on a companding technique with trapezium distribution," *IEEE Trans. Broadcast.*, vol. 57, no. 2, pp. 291–298, Jun. 2011.

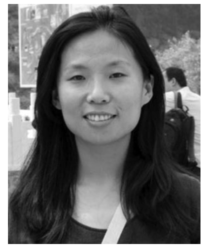
[12] Y. Wang, J. Ge, L. Wang, J. Li, and B. Ai, "Nonlinear companding transform for reduction of peak-to-average power ratio in OFDM systems," *IEEE Trans. Broadcast.*, vol. 59, no. 2, pp. 369–375, Jun. 2013.

[13] S. A. Aburakhia, E. F. Badran, and D. A. E. Mohamed, "Linear companding transform for the reduction of peak-to-average power ratio of OFDM signals," *IEEE Trans. Broadcast.*, vol. 55, no. 1, pp. 155–160, Mar. 2009.

[14] P. Yang and A. Hu, "Two-piecewise companding transform for PAPR reduction of OFDM signals," in *Proc. Wireless Commun. Mobile Comput. Conf. (IWCMC)*, Istanbul, Turkey, Jul. 2011, pp. 619–623.

[15] J. Hou, J. Ge, D. Zhai, and J. Li, "Peak-to-average power ratio reduction of OFDM signals with nonlinear companding scheme," *IEEE Trans. Broadcast.*, vol. 56, no. 2, pp. 258–262, Jun. 2010.

[16] J. J. Bussgang, "Crosscorrelation function of amplitude-distorted Gaussian signals," Res. Lab. Electron., Massachusetts Inst. Technol., Cambridge, MA, USA, Tech. Rep. 216, Mar. 1952.



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