Abstract—This paper focuses on the use of magneto-resistive and sonar sensors for imminent collision detection in cars. The magneto-resistive sensors are used to measure the magnetic field from another vehicle in close proximity, to estimate relative position, velocity, and orientation of the vehicle from the measurements. First, an analytical formulation is developed for the planar variation of the magnetic field from a car as a function of 2-D position and orientation. While this relationship can be used to estimate position and orientation, a challenge is posed by the fact that the parameters in the analytical function vary with the type and model of the encountered car. Since the type of vehicle encountered is not known a priori, the parameters in the magnetic field function are unknown. The use of both sonar and magneto-resistive sensors and an adaptive estimator is shown to address this problem. While the sonar sensors do not work at very small intervehicle distance and have low refresh rates, their use during a short initial time duration leads to a reliable estimator. Experimental results are presented for both a laboratory wheeled car door and for a full-scale passenger sedan. The results show that planar position and orientation can be accurately estimated for a range of relative motions at different oblique angles.

Index Terms—Crash detection, crash sensors, extended Kalman filter (EKF), magnetic sensors, sensor fusion, vehicle position sensors.

I. INTRODUCTION

This work in this paper is motivated by the need to develop an inexpensive sensor system for an automobile that can predict an imminent collision with another vehicle, just before the collision occurs. The prediction needs to occur at least 100 ms before the collision, so that there is adequate time to initiate active passenger protection measures to protect the occupants of the vehicle during the crash. Examples of simple occupant protection measures that can be initiated based on the prediction include pretightening of seat belts and gentler inflation of air bags. In addition, active crash space enhancement systems such as active bumpers [1], [2] and rapid active seat back control [3] can be utilized.

It should be noted that active occupant protection measures involve considerable cost, discomfort, and even a small risk to the occupants. For example, deployment of air bags is an expensive action resulting in considerable cost. Likewise, rapid seatback motion control during driving can be significant annoyance and a danger to the driver, if triggered unnecessarily. Therefore, these measures can be initiated only if the collision prediction system is highly reliable. A false prediction of collision has highly unacceptable costs.

Traditionally, radar and laser systems have been used on cars for adaptive cruise control and collision avoidance [4]–[9]. These sensors typically work at intervehicle spacing greater than 1 m. They do not work at very small intervehicle spacing and further have a very narrow field of view at small distances [6]. Collision prediction based on sensing at large distances is unreliable. For example, even if the relative longitudinal velocity between two vehicles in the same lane is very high, one of the two vehicles could make a lane change resulting in no collision. An imminent collision can be reliably predicted enough to inflate air bags only when the distance between vehicles is very small and when it is clear that the collision cannot be avoided under any circumstances. Radar and laser sensors are not useful for such small distance measurements. A radar or a laser sensor can cost well over $1000. Hence, it is also inconceivable that a number of radar and laser sensors be distributed all around the car in order to predict all the possible types of collisions that can occur. It should be noted that camera-based image processing systems suffer from some of the same narrow field of view problems for small distances between vehicles.

Therefore, this paper focuses on the development of a sensor system that can measure relative vehicle position, velocity, and orientation at very small intervehicle distances. The main idea of the new proposed sensing system is to use the inherent magnetic field of a vehicle for position estimation. A vehicle is made of many metallic parts (for example, chassis, engine, body, etc.), which have a residual magnetic field and/or get magnetized in the Earth’s magnetic field. These magnetic fields create a net magnetic field for the whole vehicle, which can be analytically modeled as a function of vehicle-specific parameters and position around the vehicle. By measuring the magnetic field using anisotropic magneto-resistive (AMR) sensors, the position of the vehicle can be estimated if the vehicle-specific parameters are known. However, the specific parameters vary from one type of vehicle to another type of vehicle, both due to varying amounts of ferromagnetic material and varying levels of magnetization. Hence, these parameters need to be estimated along with the position for every encountered vehicle. This challenge is addressed by the use of both sonar and magneto-resistive sensors and an adaptive estimator. While the sonar sensors do not work at very small intervehicle distances...
and have low refresh rates, their use during a short initial time duration leads to a reliable estimator. Magnetoresistive sensors offer the advantages of being able to work at very small distances, down to 0 m, having a very high refresh rate, and being highly inexpensive and compact. To the best of the authors’ knowledge, this is the first journal publication that utilizes magnetoresistive sensors for 2-D vehicle position estimation.

An AMR sensor has a silicon chip with a thick coating of piezoresistive nickel–iron. Every car has a magnetic field, which is a function of the vehicle position around the vehicle. The presence of an automobile in close range to an AMR sensor causes a change in the magnetic field, which changes the resistance of the nickel–iron layer. By measuring the resistance, obtaining the real-time magnetic field, and developing models that relate magnetic field to intervehicle position, estimation systems for intervehicle distance estimation can be developed. The HMC2003 three-axis magnetic sensor boards from Honeywell are utilized for the system developed in this paper. Each sensor board contains core HMC100x AMR sensing chips, which cost about $10. Application note AN218 from Honeywell describes the use of the AMR chips for vehicle detection and traffic counting applications (neither of which involves vehicle position estimation) [10].

A custom-designed sonar system is also used, which consists of one transmitter and two receivers, and measures not only the distance to the objects but also the orientation of the object. This system is described in detail in later sections.

This paper is organized as follows. In Section II, a brief review of authors’ previous work on 1-D position estimation using AMR sensors is provided. In Section III, the problem of 2-D vehicle position estimation and the challenges in using magnetic field for that application are discussed. This discussion is followed by the derivation of the 2-D magnetic field equations for a car presented in Section IV. The custom-designed sonar system that has been developed is discussed in Section V. The extended Kalman filter (EKF) used for 2-D position estimation adopting measurements from the magnetic sensors and the sonar sensor is discussed in Section VI. Results of different experiments are presented in Section VII. Finally, this paper is concluded in Section VIII. Thus, this paper develops the magnetic field equations for a car, develops an EKF for relative position and orientation estimation in close proximity, based on measurements from magnetic sensors and a custom-designed sonar system, and verifies the performance of the developed estimator through different experiments.

II. ONE-DIMENSIONAL POSITION ESTIMATION

Every car has a magnetic field, which is a function of the position around the vehicle. This was demonstrated earlier purely for 1-D vehicle position estimation in [11]. This paper considers 2-D position estimation, building on a previous conference publication by the same authors [12]. The parameters in the magnetic field versus position function vary with the type and model of encountered vehicle. Since the type of vehicle encountered is not known a priori, the parameters of the magnetic field function are unknown. Therefore, adaptive estimation algorithms are used for estimation of 2-D position and orientation from magnetic field measurements.

For 1-D case, a number of tests with different vehicles were first performed in order to experimentally investigate the magnetic field generated by an encountered vehicle as a function of 1-D distance. Fig. 1 shows a general schematic of the tests. An AMR sensor and a sonar sensor were packaged on a printed circuit board (PCB) together with a microprocessor (see Fig. 2) that reads the sensors signals and transmits their values to a computer.

The outputs of the AMR and sonar sensors were sampled at the rate of 2 kHz using a dsPIC microcontroller with a 12-bit analog-to-digital converter (ADC). Fig. 3 shows the relationship between the magnetic field (in the X-direction) and the actual distance obtained from a sonar sensor for a typical test using a Volkswagen Passat vehicle. Magnetic field is plotted in arbitrary voltage units, the same as what was read from the ADC of the microcontroller. It can be seen that there is obviously a nonlinear relation between the measured magnetic field and distance.

Based on the preceding experimental data and an analytical expression developed in [11], it was observed that below a threshold distance, i.e., $x_{th}$, the following relation holds between magnetic field and distance:

$$B = \frac{p}{x} + q$$

(1)

where $B$ is the magnetic field, $x$ is the distance of the vehicle from the sensors, and $p$ and $q$ are the vehicle-dependent parameters. This equation was fit to experimental data from various vehicles. Fig. 3 shows the fitting results from an experiment with a Volkswagen Passat vehicle. The vehicle was moved from an initial distance toward the sensors. In Fig. 3, data set 1 is the set of data points obtained after a certain time when the vehicle gets closer than $x_{th}$ to the sensors. This data set was used for curve fitting. Data set 2 is the set of data points from...
the same experiment where the vehicle is farther than $x_{th}$ from the sensors and plotted for comparison.

The equation was also verified against data from the same type of experiment with a Chevrolet Impala, a Hyundai Elantra, and a Honda Accord vehicle. Table I summarizes the results of the experiments showing the coefficient of determination ($R^2$) of the fitted line and estimated $x_{th}$ for various vehicles.

The next step would be to adopt the proposed equation for proximity sensing. However, the values of the parameters $p$ and $q$ vary with the vehicle, being constant for a specific vehicle but changing from one model to another. Since the type of vehicle encountered is not known a priori, these parameters have to be adaptively updated in real time. This has been discussed in [11].

### III. Two-Dimensional Position Estimation

For 1-D motion, in which the vehicle is moving directly toward or away from the sensors, we found that, below a threshold distance $x_{th}$, the relation in (1) can be assumed between magnetic field and distance. That relationship could be used together with an adaptive estimator for real-time estimation of position just prior to a frontal or rear-end collision. However, an impact due to collision can occur at any location around the car body. In fact, side impact and oblique collisions at rural intersections are a significant source of fatalities [13]. It is therefore necessary to be able to estimate not only the relative position but also the orientation of the colliding vehicle anywhere in the 2-D plane.

To further investigate the 2-D motion problem, first consider the simplified case in which the vehicle is moving toward the sensors at a constant angle $\theta$, as shown in Fig. 4. In this case, if the angle $\theta$ is known, the magnetic field along the direction of motion of the vehicle can be expressed in terms of the AMR sensor’s measurements as

$$B = B_x \cos \theta + B_y \sin \theta = \frac{p}{r} + q$$

(2)

where $r$ is the distance measured along the direction of motion. This equation can potentially be used with an adaptive algorithm to estimate $r$.

However, if $\theta$ is not constant or if the colliding vehicle is moving toward the sensors at an offset (meaning that its center line does not pass through the center of the AMR sensor), the preceding approach cannot be adopted.

Hence, to fully identify and classify a crash in 2-D motion, we need to estimate $x_A$, $y_A$, $v$, $\theta$, and $\omega$, as shown in Fig. 5, where $x_A$ and $y_A$ are the position of point $A$ with respect to the coordinate frame attached to the approaching car, $v$ is the longitudinal velocity of the approaching car along its $x$-axis, $\theta$ is the orientation of the approaching car relative to the host car (in other words, it is the angle between the $x$-axis of the coordinate frame attached to the approaching car and the $X$-axis of the coordinate frame at point $A$), and $\omega$ is the rotational velocity of the approaching car.

It is worth mentioning that another way of expressing the position of the objects would be to express the position of point $O$ with respect to the coordinate frame attached to point $A$. However, using this coordinate frame, the measurement equations, which are derived later on, will be more complicated.

Use of magnetic field for estimation of vehicle position in the 2-D motion is much more complicated than in the 1-D motion not only because of the additional degree of freedom for the vehicle but also because of the complex pattern of the vehicle’s magnetic field in 2-D space. The magnetic field lines are not parallel to each other, and they curve out to the sides. This is

**TABLE I**

| Vehicle       | $|p|$ | $B_{stat} - q$ | $R^2$ | $x_{th}$ |
|---------------|------|---------------|-------|----------|
| Chevy Impala  | 25.26| 3.23          | 0.997 | 4.8      |
| Honda Accord  | 28.42| -6.79         | 0.999 | 3.2      |
| VW Passat     | 74.38| 14.38         | 0.997 | 4.5      |
| Hyundai Elantra| 10.2 | -3.21         | 0.999 | 3        |

![Fig. 3. Results of the experiment with a Volkswagen Passat vehicle and the fitted curve.](image)

![Fig. 4. Vehicle moving toward sensors at a constant angle.](image)

![Fig. 5. Two-dimensional position estimation and the parameters to be estimated.](image)
the same with any type of magnetic objects. Fig. 6 shows the magnetic field lines of a rectangular magnet.

This phenomenon can be also observed in the experiments with the door of a Ford passenger sedan. The door is mounted on a wheeled platform, as shown in Fig. 7, and can be maneuvered and moved easily toward or away from the sensors in different directions.

Fig. 8 shows a case where the door is coming toward a set of four AMR sensors at a 45° angle. The sensor readings at four different locations of the door during its motion are shown. As shown in the figure, initially, the direction of the magnetic field (shown with a green dashed line) is very different from the normal direction of the door. Hence, it is not possible to obtain the orientation of the door, i.e., \( \theta \), by only determining the direction of the magnetic field. It can be also seen that as the door gets closer, the magnetic field magnitude increases, and its direction changes to become in line with the normal direction of the door.

To further investigate a vehicle’s magnetic field, a mathematical expression for the field in a 2-D plane created by a rectangular magnetic body is derived in the next section.

IV. DERIVATION OF A MATHEMATICAL EXPRESSION FOR MAGNETIC FIELD IN TWO DIMENSIONS

Modeling a vehicle as a rectangular block of magnetic dipoles, as shown in Fig. 9, we want to obtain a mathematical expression for the magnetic field at an arbitrary point \( A \). While such a magnetic field approximation for a vehicle body might at first appear crude, the model obtained from such an assumption, together with parameter estimation, is likely to be significantly more useful than a purely empirical function obtained without any magnetic field models. The goal is to later use the derived magnetic field equations and estimate vehicle position by measuring \( B_x \) and \( B_y \) using an AMR sensor at point \( A \). In the following derivations, the height of the block \( da \) is assumed to be small with respect to its width and length.

As a first step, the magnetic field of a line of magnetic dipoles (see Fig. 10) is obtained.

According to [14], the following relations can be written down:

\[
\begin{align*}
\frac{dB_r}{r^3} &= \frac{\mu_0 d m_0}{2 \pi r^3} \cos \alpha \\
\frac{dB_\alpha}{r^3} &= \frac{\mu_0 d m_0}{4 \pi r^3} \sin \alpha
\end{align*}
\]

In order to obtain \( B_x \) and \( B_y \), we need to integrate \( dB_x \) and \( dB_y \), which are given by the following equations:

\[
\begin{align*}
B_x &= dB_r \cos \alpha - dB_\alpha \sin \alpha \\
B_y &= dB_r \sin \alpha + dB_\alpha \cos \alpha
\end{align*}
\]
Expressing $r$ and $\alpha$ in terms of $x$ and $y$, we have

$$r = (x^2 + (y_A - y)^2)^{\frac{1}{2}}$$

$$\alpha = \tan\left(\frac{y_A - y}{x_A}\right)$$

$$\sin\alpha = \frac{y_A - y}{(x_A^2 + (y_A - y)^2)^{\frac{1}{2}}}$$

$$\cos\alpha = \frac{x_A}{(x_A^2 + (y_A - y)^2)^{\frac{1}{2}}}.$$  

Hence

$$dB_x = \frac{\mu_0 dm_0}{4\pi r^3} (2\cos^2 \alpha - \sin^2 \alpha)$$

$$= \frac{\mu_0 dm_0}{4\pi} \frac{2x_A^2 - (y_A - y)^2}{(x_A^2 + (y_A - y)^2)^{\frac{3}{2}}}$$

$$dB_y = \frac{\mu_0 dm_0}{4\pi r^3} 3\cos\alpha \sin\alpha$$

$$= \frac{\mu_0 dm_0}{4\pi} \frac{3x_A(y_A - y)}{(x_A^2 + (y_A - y)^2)^{\frac{3}{2}}}.$$  

The magnetic field generated by the line of magnetic dipoles is then obtained by integrating the preceding terms as

$$B_{x-l}(x_A, y_A) = \frac{\mu_0 m_0 dx}{4\pi} \int_{-b}^{b} \frac{2x_A^2 - (y_A - y)^2}{(x_A^2 + (y_A - y)^2)^{\frac{3}{2}}} dy$$

$$= \frac{\mu_0 m_0 dx}{4\pi} \left( \frac{2x_A^2(y_A + b) + (y_A + b)^3}{x_A^2(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. - \frac{2x_A^2(y_A - b) + (y_A - b)^3}{x_A^2(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right)$$

$$B_{y-l}(x_A, y_A) = \frac{\mu_0 m_0 dx}{4\pi} \int_{-b}^{b} \frac{3x_A(y_A - y)}{(x_A^2 + (y_A - y)^2)^{\frac{3}{2}}} dy$$

$$= \frac{m_0 dx}{4\pi} \left( \frac{x_A}{x_A^2(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. + \frac{x_A}{x_A^2(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right).$$

In order to obtain the total magnetic field created by the whole bar, $B_{x-l}$ and $B_{y-l}$ can be integrated in the $X$-direction as

$$B_x = \int_{x_A}^{x_A + L} B_x(x_A, y_A)$$

$$= \frac{\mu_0 m_0 da}{4\pi} \left( \frac{y_A + b}{(x_A + L)^2(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. - \frac{y_A - b}{x_A(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right)$$

$$B_y = \frac{\mu_0 m_0 da}{4\pi} \left( \frac{1}{(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. - \frac{1}{(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right).$$

The equations for the magnetic field derived in (13) are complex and not suitable for real-time estimation of both equation parameters and position. However, it is possible to simplify (13) for the colored region shown in Fig. 11 for which we have $y_A \ll b$ and $x_A \leq x_{th}$.

Assuming $x_A + L \gg x_A$, (13) can be simplified to

$$B_x = \frac{\mu_0 m_0 da}{4\pi} \left( \frac{y_A + b}{x_A(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. - \frac{y_A - b}{x_A(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right)$$

$$B_y = \frac{\mu_0 m_0 da}{4\pi} \left( \frac{1}{(x_A^2 + (y_A + b)^2)^{\frac{3}{2}}} \right.$$

$$\left. + \frac{1}{(x_A^2 + (y_A - b)^2)^{\frac{3}{2}}} \right).$$
Next, if we assume that \( y_A \) is small and close to zero, (14) can be further simplified to the following equations:

\[
B_x = \frac{\mu_0 m_0 da}{2\pi} \frac{b}{x_A (x_A^2 + b^2)^{\frac{3}{2}}} + f(p, b, x_A) y^2 + \cdots \\
= \frac{\mu_0 m_0 da}{2\pi} \frac{b}{x_A (x_A^2 + b^2)^{\frac{3}{2}}} \tag{15}
\]

\[
B_y = \frac{\mu_0 m_0 da}{2\pi} \frac{by}{(x_A^2 + b^2)^{\frac{3}{2}}} + f(p, b, x_A) y^3 + \cdots \\
= \frac{\mu_0 m_0 da}{2\pi} \frac{by}{(x_A^2 + b^2)^{\frac{3}{2}}} \tag{16}
\]

It should be noted that (15) is the same as the equation obtained earlier for the 1-D case, replacing \( da \) with \( 2a \). For small values of \( x_A \), the preceding equations can be simplified further as

\[
B_x = \frac{\mu_0 m_0 da}{2\pi} \frac{1}{x_A (1 + (\frac{b}{x_A})^2)^{\frac{3}{2}}} \approx \frac{\mu_0 m_0 da}{2\pi x_A} = \frac{p}{x_A} \tag{17}
\]

\[
B_y = \frac{\mu_0 m_0 da}{2\pi} \frac{by}{(x_A^2 + b^2)^{\frac{3}{2}}} = \frac{pby}{(x_A^2 + b^2)^{\frac{3}{2}}} \tag{18}
\]

If there exists any static magnetic field at point \( A \), like the Earth’s magnetic field, a constant needs to be added to the preceding equations to obtain the total magnetic field to be measured by the sensors. Thus

\[
B_x = \frac{p}{x_A} + q_x \tag{19}
\]

\[
B_y = \frac{pby}{(x_A^2 + b^2)^{\frac{3}{2}}} + q_y. \tag{20}
\]

It is also possible to subtract the static magnetic field at the location of the AMR sensors from the measurements to avoid adding a constant to readings from the AMR sensors. However, for readings of \( B_x \), it was observed that, even when subtracting the static magnetic field from the measurements, (19) results in a better fit compared with using (17). Therefore, the following equations are being used for position estimation in the next sections of this paper:

\[
B_x = \frac{p}{x_A} + q \tag{21}
\]

\[
B_y = \frac{pby}{(x_A^2 + b^2)^{\frac{3}{2}}}. \tag{22}
\]

In order to use (21) and (22) for position estimation by measuring the magnetic field, we need to make sure that the AMR sensors are in the colored region shown in Fig. 11 for which we have simplified the magnetic field equations. For the points inside this region, the ratio \( |B_y|/|B_x| \) is small. Hence, we can install multiple AMR sensors around the host vehicle and check the ratio \( |B_y|/|B_x| \) for each AMR sensor and pick the sensors with the lowest ratio as the most appropriate sensor to utilize for position estimation. This concept is shown in Fig. 12.

Each AMR sensor measures the magnetic field along its \( X \) and \( Y \)-axes, i.e., \( B_{xAMR} \) and \( B_{yAMR} \). Hence, the magnetic field of the approaching vehicle at the location of AMR sensor \( i \) expressed in the approaching vehicle coordinate frame can be obtained as

\[
\begin{align*}
B_{xi} &= B_{xAMRi} \cos(\theta) + B_{yAMRi} \sin(\theta) \\
B_{yi} &= -B_{xAMRi} \sin(\theta) + B_{yAMRi} \cos(\theta). \tag{23}
\end{align*}
\]

As can be seen from (23), to calculate the ratio \( |B_{yi}|/|B_{xi}| \) in order to select the most appropriate AMR sensors for measurement update, we need at least an initial estimate of \( \theta \). Therefore, a custom-designed sonar system has been developed, which measures both the distance to the object as well as its orientation. The sonar sensor detects the objects at longer distances compared with AMR sensors. Hence, when the AMR sensors respond to the presence of a car, the estimated \( \theta \) from the sonar system can be used to select the most appropriate AMR sensors for measurement updates. The sonar sensor is also used to initialize the magnetic field parameters, and this is shown to speed up the convergence of the magnetic field parameters. The sonar system is described in the following section.

V. SONAR MEASUREMENT SYSTEM

The developed sonar measurement system includes one transmitter, i.e., \( T \), at point \( A \) and two receivers, i.e., \( R_1 \) and \( R_2 \), at distances \( d_1 \) and \( d_2 \) from \( A \) arranged in the order shown in Fig. 13. This configuration of the transmitter and the receivers makes it possible to measure the orientation of the target and its distance from the sensors.

Measuring the travel time of sound for receivers 1 and 2, the distance that the echo pulse has traveled can be calculated. Using the fact that the incident and reflected angles of sound are the same (similar to light when it reflects from a mirror), it can be concluded that the measured distances equal \( l_1 \) and \( l_2 \), as shown in Fig. 13. In other words, \( l_1 \) and \( l_2 \) equal the distance from the image of transmitter \( T \) at point \( B \) to the receivers \( R_1 \) and \( R_2 \), respectively.

Knowing \( l_1 \) and \( l_2 \) and the distance between the transmitter and the receivers, i.e., \( d_1 \) and \( d_2 \), the angle \( \theta \) can be calculated using the cosine rule as

\[
d_\theta = d_1 + d_2 \tag{24}
\]
他知道\( \theta_2 \), the length \( l_s \), i.e., the distance between the transmitter and its image \( (AB) \), can be calculated using the cosine rule as

\[
l_s^2 = d_2^2 + l_s^2 - 2d_2l_s \cos(90 - \theta_2). \tag{27}
\]

Then \( x_s \) \((AC)\), i.e., the sonar estimate of \( x_A \), can be calculated from \( l_s \) since

\[
x_s = \frac{l_s}{2}. \tag{28}
\]

Applying the cosine rule one more time, \( \theta_s \), i.e., the sonar estimate of \( \theta \), can be also calculated as

\[
l_s^2 = d_2^2 + l_s^2 - 2d_2l_s \cos(90 + \theta_s). \tag{29}
\]

In practice, the measured signals from the sonar sensors are \( l_{1m} \) and \( l_{2m} \), where

\[
l_{1m} = l_1 + n_1 \quad n_1 \sim N(0, \sigma_s)
\]

\[
l_{2m} = l_2 + n_2 \quad n_2 \sim N(0, \sigma_s).
\]

If we want to use \( x_s \) and \( \theta_s \) in the estimator, we should also calculate the covariance of noise in the measurements. Since \( x_s \) and \( \theta_s \) have nonlinear relations with \( n_1 \) and \( n_2 \), we need to calculate the derivatives of \( x_s \) and \( \theta_s \) with respect to \( n_1 \) and \( n_2 \), which would be

\[
\frac{\partial x_s}{\partial n_1} = -\frac{l_{1m}d_2}{2l_s d_s}, \tag{30}
\]

\[
\frac{\partial x_s}{\partial n_2} = -\frac{l_{2m}d_1}{2l_s d_s}. \tag{31}
\]

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\frac{\partial \theta_s}{\partial n_1} = \frac{\cos(y_2) l_{1m} l_{2m}}{\cos(\theta_s) d_s l_s^2} \tag{32}
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\frac{\partial \theta_s}{\partial n_2} = \frac{\cos(y_1) l_{1m} l_{2m}}{\cos(\theta_s) d_s l_s^2}. \tag{33}
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Similar to any sonar sensor used for 1-D position measurements, the developed sonar system will not also work at close proximities (typically below 0.5 m). In addition, the sonar system will not work if the orientation of the object, i.e., \( \theta \), increases so that \( \theta_1 \) and \( \theta_2 \) are greater than the beam angles of the transducers. It should be mentioned that the threshold for \( \theta \) is not constant and slightly changes with the distance from the object. Hence, there cannot be a constant value for \( \theta_{\text{threshold}} \). However, an easier way of detecting outliers is at the beginning of the calculation, by looking at the values obtained for \( \theta_1 \) and \( \theta_2 \). If they have imaginary values, then an outlier is detected. This method for detecting outliers is described later.

It is also worth mentioning here that the sonar system alone is not able to determine \( y_A \). In other words, the object can move arbitrary along its \( Y \)-axis and the sonar readings will remain the same. Finally, the sonar system has a much lower refresh rate (20 Hz) compared with AMR sensors (1 Hz). Hence, the use of the AMR sensors is critical.

Addressing Crosstalk and Multipath Problems of the Sonar System: Problems with crosstalk and multipath returns are addressed in three different layers of the proposed system: design level, sonar sensor level, and estimator level.

1) Design level: The sonar transmitter is set to send out a signal every 50 ms, and each receiver hears back the first signal that returns to it. After the first signal has been detected, the receivers ignore any returning signals until the next signal is transmitted. The voltage level of the signal sent to the transmitter and the time duration of 50 ms are design parameters. The voltage level is set such that the sonar system is not able to detect objects more than \( \sim 6 \) m away. Since the speed of sound is about 340 m/s, all the possible returns are within \( \text{dt} = 2 \times 6/340 = 35 \) ms of the transmitter. Hence, transmitting signals every 50 ms ensures that there will not be any returns from a previous transmit to interfere with the current measurement.

2) Sonar sensor level: The designed sonar system includes one transmitter and two receivers. Using the two receivers, it is possible to measure the distance to the object as well as its orientation. On the other hand, having two receivers provides another level of robustness. As an example, assume that there is an object at a distance of \( x_A = 2 \) m in front of the sonar system at an angle of \( \theta = 20^\circ \). The raw readings from the two receivers will be \( l_1 = 3.95 \) m and \( l_2 = 4.05 \) m, from which \( x_s \) and \( \theta_s \) can be calculated as described by (24)–(29). Now, let us assume that due to a multipath or a crosstalk error, \( l_1 \) is measured as \( l_1' = 3 \) m. This will cause the argument of the “asin” function in (26) to be larger than 1, causing an imaginary value for \( \theta_2 \). Hence, in general, by looking at the values obtained for \( \theta_1 \) and \( \theta_2 \), we can determine if the readings are valid or not. This adds another level of robustness against multipath and crosstalk errors.
3) Estimator level: Finally, in the estimator, the “Mahalanobis distance” is used to reject false readings of the sonar sensing system. The Mahalanobis distance is defined as

$$d_{\text{mah}_-k} = \left( (x_{s-k} - \hat{x}_{k}) (P_{x-k}^{-1}) (x_{s-k} - \hat{x}_{k}) \right)^{0.5}$$

(34)

where $x_{s-k}$ is the measured distance from the sonar system, $\hat{x}_{k}$ is the a priori estimated distance, and $P_{x-k}$ is the a priori covariance of the estimated distance. For a given $P_{x-k}$, the farther the measurement is from the predicted estimate, the larger is the Mahalanobis distance. For a certain distance between the measurement and the predicted estimate, the smaller the uncertainty in the predicted estimate is, the larger is the Mahalanobis distance. In other words, as we are more certain about the current estimate, we reject a measurement with a shorter distance to the predicted estimate compared with the case when we are less certain about the current estimate and we allow a larger distance between the measurement and the estimated distance. Fig. 21 in Section VII shows how the Mahalanobis distance is used to reject sonar outliers in the experiment with the vehicle door.

VI. EKF FOR TWO-DIMENSIONAL POSITION AND ORIENTATION ESTIMATION

As mentioned earlier, the current 2-D crash prediction system is based on four AMR sensors and a sonar system consisting of three sonar transducers, as shown in Fig. 14.

The sonar sensing system works at larger distances compared with AMR sensors; however, it does not work at short distances. To account for the different working ranges of the sensors, a state machine shown in Fig. 15 is utilized. In state 0, the estimator will use the sonar sensor to update position since the AMR sensors are not yet affected by the approaching vehicle. As soon as the AMR sensors respond to the approaching vehicle, updates would be done using both sonar and AMR sensors; however, it does not work at short distances. As mentioned earlier for the magnetic field in two dimension without any simplifying assumptions. For instance, it is possible to use equations obtained earlier for the magnetic field in two dimension without any simplifying assumptions. However, calculating the Jacobian required for EKF measurement update will become complicated and computationally intense. It is also worth mentioning that the ordinary unscented Kalman filter [17], [18] fails in this problem mainly because of the discontinuity at $x = 0$.

With the aforementioned explanations, we will derive the equations for the EKF. It should be noted that the equations presented here are the general equations used in state 1, where both sonar and AMR sensor measurements are available. In case of states 0 or 2, the Kalman filter measurement equations to state 1, but there is no prior knowledge about $x_{th}$ when a new vehicle is approaching. Therefore, the covariance of the AMR data at predetermined time intervals can be used instead. Starting from state 0, whenever the covariance is higher than a threshold, the estimator switches to state 1 in order to switch from sonar to sonar–AMR updates. To obtain more meaningful initial values for the states $p$ and $q$, a least squares (LS) fitting can be performed at the switching time. The estimated values and their covariance are used as initial values for $p$ and $q$ and their covariance. While in state 1, $x_{th}$ can be calculated in real time and used for determining if the vehicle is moving out of the view of the AMR sensors or the sonar sensor and if the system should switch back to state 0 or switch to state 2.

The new sonar system also measures the orientation of the object, i.e., $\theta$; and if $\theta$ increases beyond a threshold, it will not be able to measure the orientation due to the limitations in the beam width of the sonar transducers. This also results in a transition from state 1 to state 2.

The EKF [15], [16] is used for estimation of magnetic field equation parameters, as well as position, orientation, and velocity of the approaching vehicle. There are some tight time constraints with the real-time 2-D positioning system. Sensors data are captured through two microchip dsPIC microcontrollers and transferred to MATLAB running on a PC at 500 Hz via serial port. Since the system is working in real time, there would be a time period of 2 ms for transferring data from microcontrollers to MATLAB (taking about 0.5 ms), analyzing data, and visualization. Having an EKF running in MATLAB (described in later sections), it means that there is about 1.5 ms to perform time and measurement update steps of the EKF. Therefore, the measurement equations should be simplified as much as possible. For instance, it is possible to use equations obtained earlier for the magnetic field in two dimension without any simplifying assumptions. However, calculating the Jacobian required for EKF measurement update will become complicated and computationally intense. It is also worth mentioning that the ordinary unscented Kalman filter [17], [18] fails in this problem mainly because of the discontinuity at $x = 0$. With the aforementioned explanations, we will derive the equations for the EKF.
will be a reduced version of the measurement equations derived here. The state vector to be estimated is

$$X = [x \ y \ v \ \alpha \ \theta \ \omega \ \alpha \ p \ q]^T$$  \hspace{1cm} (35)$$

where $x$, $y$, and $\theta$ express the position and orientation of the object; $v$ and $\omega$ are the longitudinal and rotational velocities of the object, respectively; $\alpha$ and $\alpha$ are the longitudinal and rotational accelerations, respectively; and $p$ and $q$ are the magnetic field equation constants.

A. Time Update Equations

In order to obtain the time update equations of the EKF, we need to derive the dynamic equations of the system. Considering Fig. 5, it is assumed that the object moves with a longitudinal velocity $v$ along its $x$-axis and a rotational velocity $\omega$. Using the transport theorem, the dynamic equations can be written down as

$$\dot{r}_A = (\dot{r}_A)_\text{rel} + \omega \times r_A \Rightarrow \dot{x}_A = -v + \omega y_A & \dot{y}_A = -\omega x_A.$$  \hspace{1cm} (36)$$

Discretizing the preceding equations, dropping the $A$ subscript, and including longitudinal and rotational accelerations, the following equations for the dynamics of the system are obtained:

$$X_k = f(X_{k-1}, w_{k-1}), \quad w_k \sim (0, Q_k)$$

$$x_k = x_{k-1} - v_{k-1} dt + \omega_{k-1} y_{k-1} dt$$

$$y_k = y_{k-1} - \omega_{k-1} x_{k-1} dt$$

$$a_k = a_{k-1} + w_{k-1}^3$$

$$\omega_k = \omega_{k-1} + \alpha_{k-1} dt$$

$$p_k = p_{k-1} + w_{k-1}^4$$

$$q_k = q_{k-1} + w_{k-1}^4.$$  \hspace{1cm} (37)$$

The time update equations will be as in (38), shown at the bottom of the page.

Next, the measurement update equations of the EKF are derived.

B. Measurement Update Equations

Here, the measurement update equations for state 1 are derived. States 0 and 2 have a reduced version of the measurement update equations derived here. In state 1, the available measurements are the distance $x_s$ and the orientation $\theta_s$ from the sonar system and eight measurements from four AMR sensors $B_{xAMR i}$, $B_{yAMR i}$ ($i = 1, 2, 3, 4$), which are measured with respect to the coordinate frame of each AMR sensor. At each measurement update, first, the measured magnetic fields by each AMR sensor expressed in the coordinate frame of the approaching vehicle, i.e., $B_{xi}$ and $B_{yi}$, are calculated as

$$B_{xi} = B_{xAMR i} \cos \theta \hat{e}_k + B_{yAMR i} \sin \theta \hat{e}_k$$  \hspace{1cm} (39)$$

$$B_{yi} = -B_{xAMR i} \sin \theta \hat{e}_k + B_{yAMR i} \cos \theta \hat{e}_k, \quad i = 1, 2, 3, 4.$$  \hspace{1cm} (40)$$

Second, the ratio between the magnetic fields in $Y$- and $X$-directions, i.e., $B_{RAT i} = |B_{yi}/B_{xi}|$, is calculated for each sensor. Then, the two sensors that have lower values of $B_{RAT}$ are selected (named $m$ and $n$), and the corresponding values of $B_{xi}$ and $B_{yi}$ of those two sensors are assigned to the new $B_{xi}$ and $B_{yi}$, respectively. The result will be the measurement update equations in (41) and (42), shown at the bottom of the next page, where $x_i = x + d_i \cos \theta$, i.e., $i = m, n; d_m$ and $d_n$ are the distances of AMR sensors $m$ and
where the following relations:

\[
\begin{align*}
B_{RAT} & = B_x = \frac{y x_A}{x_A^2 + b^2}, \\
B_{m} & = \frac{\mu_0 m_0 d a}{2\pi x_A (x_A^2 + b^2)^{2}}, \\
B_{y} & = \frac{\mu_0 m_0 d a}{2\pi y b (x_A^2 + b^2)^{2}}, \\
B_{RAT} & = B_x = \frac{y x_A}{x_A^2 + b^2}, \\
\end{align*}
\]

(43)

For each one of the selected sensors, i.e., \( m \) and \( n \), we have the following linear equations:

\[
\begin{align*}
B_{RATm} T_m - y_m (\hat{x}_k + d_m \sin \hat{\theta}_k) & = 0, \\
B_{RATn} T_n - y_n (\hat{x}_k + d_n \sin \hat{\theta}_k) & = 0, \\
y_m - y_n & = (d_m - d_n) \cos \hat{\theta}_k, \\
T_m - T_n & = (\hat{x}_k + d_m \sin \hat{\theta}_k)^2 - (\hat{x}_k + d_n \sin \hat{\theta}_k)^2, \\
\end{align*}
\]

(45)

where

\[
T_i = (\hat{x}_k + d_i \sin \hat{\theta}_k)^2 + b^2, \quad i = m, n.
\]

(46)

Solving for \( y_m, y_n, T_m, \) and \( T_n \), we can obtain an estimation of \( y \) from the following equation:

\[
y_{\text{meas}} = y_m - d_m \cos \hat{\theta}_k.
\]

(47)

It should be noted that, while calculating the Jacobian for the EKF measurement update, the effect of uncertainties in \( \hat{x}_k \) and \( \hat{\theta}_k \) on \( y_{\text{meas}} \) is ignored, and \( y_{\text{meas}} \) is assumed to have a zero mean Gaussian noise to make the Jacobian simpler.

VII. Experimental Results

The developed estimator was tested with the Ford vehicle door shown in Fig. 6 and a Mazda Protégé 1999 sedan. Testing with the door has the advantage that more complicated scenarios can be implemented since it is easier to move it around.

A. Results From the Tests With a Ford Door

The estimator was first verified with tests using the Ford vehicle door. The door is mounted on a wheeled platform, as shown in Fig. 7, and can be maneuvered and moved easily toward or away from the sensors in different angles.

Fig. 16 shows snapshots of the estimated position of the door at different time intervals during an experiment. The color of the line used to represent door position goes from black to gray from the beginning toward the end of the experiment. The red rectangles show the position of the AMR sensors (sonar transducers are not shown). The door has been moved toward the sensors from point A to point B and then rotated to \( \theta = -35^\circ \) at point B, moved closer to the sensors to point C, moved back to point B and rotated to \( \theta = 0^\circ \), and, finally, moved closer to the sensors to point D.

\[
\begin{align*}
Z & = h(X, n) \\
Z & = [x_s \theta_s 0 0 y_{\text{meas}}]^T \\
h(X, n) & = \left[ x + n_x \theta + n_\theta B_{x_n} - \left( \frac{p}{x_m} + q \right) + \left( n_{xAMR} \cos \theta + n_{yAMR} \sin \theta \right) \ldots B_{zn} - \left( \frac{p}{x_n} + q \right) + \left( n_{xAMR} \cos \theta + n_{yAMR} \sin \theta \right) y + n_y \right]^T \\
K_k & = P_k^{-} H_k^{T} \left( H_k P_k^{-} H_k^{T} + R_k \right)^{-1} \\
\hat{X}_k & = \hat{X}_k + K_k \left[ Z_k - h_k \left( \hat{X}_k, 0 \right) \right] \\
p_k^{+} & = (I - K_k H_k) P_k^{-} \\
H_k & = \frac{\partial h}{\partial X} \bigg|_{\hat{X}_k} = \left[ \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{p}{x_m} & 0 & 0 & 0 & B_{ym} + \frac{pd_m \cos \theta}{x_m} & 0 & 0 & -\frac{1}{x_m} \\
-\frac{p}{x_n} & 0 & 0 & 0 & B_{yn} + \frac{pd_n \cos \theta}{x_n} & 0 & 0 & -\frac{1}{x_n} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \\
R_k & = \text{diag}([\sigma_{x_s} \sigma_{\theta_s} \sigma_B \sigma_B \sigma_{y_{\text{meas}}}]) \\
\end{align*}
\]

(42)
Figs. 17–20 show some of the estimated states over time. It is shown in Figs. 17 and 19 that, when the door rotates to more than a threshold angle, at $t \approx 11$ s, the sonar measurements for distance and orientation are not valid anymore, and the estimator switches from state 1 to state 2 and only updates with the AMR sensors. When the door rotates back toward $\theta = 0$, the sonar measurements become valid again, at $t \approx 17$ s, and the estimator switches to state 1. In addition, when later in the experiment, at $t \approx 22$ s, the door gets very close to the sensors, the estimator switches to state 2. Also, from Fig. 18, we can see that, at the beginning of the experiment, since the AMR sensors are not available yet, updates are performed only with sonar sensors (State 0), and hence, an estimate of $y$ is not available. However, as soon as the AMR sensors become available ($t \approx 5.5$ s), estimation of $y$ becomes possible as well.

Fig. 21 shows how the Mahalanobis distance is used to reject sonar outliers in this experiment with the vehicle door. In this example, as $d_{\text{mah}} - k$ goes over the predefined threshold, the sonar measurement is ignored.

B. Results From the Tests With a Mazda Protégé Vehicle

Tests were conducted with a full-scale passenger sedan (Mazda Protégé) to evaluate the performance of the developed system at various orientation angles. In tests with the Mazda car, it was moved toward the sensors at a fixed angle. Data were obtained at various fixed angles to verify performance for different orientations. Figs. 22–26 show the result from one of the tests, where the car moved at an orientation of about $\theta = -20^\circ$ toward the sensors and then back away from the sensors. In these figures, the pink triangles indicate the time interval that a vehicle is detected and active positioning is performed. The
blue circles indicate the time interval that the AMR sensors are responding and the estimator is in state 1 or 2 depending on the availability of sonar measurements. In addition, in this test, $x_{\text{th}}$ for the sonar sensor has been set to 1 m, and hence, the estimator switches from state 1 to state 2 when $x < x_{\text{sonar-th}} = 1$ m. However, the sonar measurements are valid up to about 0.3 m from the transducers. With this higher threshold, we can evaluate the estimator performance by comparing sonar system and estimator values. The error between the AMR estimation (state 2) and the sonar measurement is shown in Fig. 26.

It should be noted that the tests here were carried out using a single vehicle. Additional vehicles could be parked close to the test vehicle if multipath and robustness issues need to be evaluated.
VIII. CONCLUSION

This paper has focused on the development of a novel and unique automotive sensor system for the measurement of relative position and orientation of another vehicle in close proximity. The sensor system is based on the use of AMR sensors, which measure magnetic field. While AMR sensors have previously been used to measure traffic flow rate and to detect vehicles in parking spots, they are used here to measure the relative 2-D position of the vehicle.

Analytical formulations were developed to predict the relationships between position and magnetic field for 2-D relative motion. The use of these relationships to estimate vehicle position is complicated by the fact that the parameters in the relationships vary with the type of vehicle under consideration. Since the type of vehicle encountered is not known a priori, the parameters for the magnetic field–position relationship have to be adaptively estimated in real time.

A system based on the use of multiple AMR sensors and a custom-designed sonar system together with an EKF was developed to estimate vehicle parameters, position, and orientation. The use of the combined sensors results in a reliable system that performs well without the knowledge of vehicle-specific magnetic field parameters. Test results with a wheeled laboratory test rig consisting of a door and tests with a full-scale passenger sedan were presented. The experimental results in this paper confirm that the developed sensor system is viable and that it is feasible to adaptively estimate vehicle position and orientation even without knowledge of vehicle-dependent parameters.

REFERENCES


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