PAPR and PICR Reduction of OFDM Signals with Clipping Noise-based Tone Injection Scheme

Jun Hou, Xiangmo Zhao, Fengkui Gong, Fei Hui, and Jianhua Ge

Abstract—Orthogonal frequency division multiplexing (OFDM) suffers from a high peak-to-average power ratio (PAPR). Tone injection (TI) extends the original constellation to several equivalent points so that these extra freedom degrees can be exploited for the PAPR reduction. However, optimal TI requires an exhaustive search over all possible constellations, which is a hard discrete optimization problem. To address this problem, a novel TI scheme that uses the clipping noise to find the optimal equivalent outer ring extended constellations and hexagonal constellations is proposed. By minimizing the mean rounding error of the clipping noise and possible equivalent constellations, the proposed scheme can easily determine the size and position of the optimal equivalent constellations. Secondly, a new TI scheme is also introduced in order to reduce the nonlinear distortion in the presence of power amplifier. This scheme takes the root-mean-square peak interference-to-carrier ratio (PICR) reduction as the target to obtain a better bit error rate (BER) performance. Simulation results show that to achieve a considerable system performance, the proposed TI schemes only need eighteen fast Fourier transforms (FFTs), while other TI and partial transmit sequence (PTS) schemes need hundreds of FFTs.

Index Terms—Orthogonal frequency division multiplexing, peak-to-average power ratio, tone injection (TI), peak interference-to-carrier ratio, power amplifier.

I. INTRODUCTION

Due to the advantages of high spectral efficiency, easy implementation with fast Fourier transform (FFT), and robustness to frequency selective fading [1], multicarrier modulation, especially orthogonal frequency division multiplexing (OFDM), has drawn explosive attention in a number of high-speed wireless communication standard systems including IEEE 802.11 a/g/n, IEEE 802.16 (WiMAX), and 3GPP LTE.

However, the high peak-to-average power ratio (PAPR) [2], [3] of the transmitted OFDM signals requires the transmit power amplifier with an extremely large dynamic range, which significantly reduces the efficiency of the amplifier. Therefore, various PAPR reduction techniques have been proposed, which can be divided in two groups. One group processes the OFDM signals directly, such as clipping and filtering method [4]-[6], companding transform [7]-[9]. In this group, the signal is clipped and filtered or companded deliberately to a predefined level, and the PAPR is reduced at the expense of the in-band distortion (e.g., bit error rate (BER) increase) and out-of-band radiation [5]. Moreover, the filtering operation may lead to the peak regrowth, which then causes the increase of PAPR.

The other group intends to reduce the occurrence of large signals before multicarrier modulation, including probabilistic techniques [10]-[13], tone reservation (TR) [14]-[16], nonbijective constellations [17]-[24], and coding [26]. Nevertheless, the complexity of the probabilistic techniques increases exponentially with the number of subblocks. Furthermore, side information may be required at the receiver to decode the input symbols. Incorrectly received side information would result in burst errors. TR also requires many iterations to ensure a sufficient PAPR reduction [14], which entails high computational complexity.

The nonbijective constellation [17] maps the data symbol to one of many constellation points. By appropriately choosing the right constellation points among the allowable set of points, the PAPR can be significantly reduced without a data rate loss or extra side information. One of effective methods of this type is tone injection (TI) [17]-[24], which uses cyclic extension of constellations to offer alternative encoding with a lower PAPR. However, implementation of the TI technique requires to solve a hard integer-programming problem, whose complexity grows exponentially with the number of subcarriers. Therefore, suboptimal TI solutions are sought [18]-[24].

In this paper, a novel TI scheme that uses the clipping noise to find the optimal equivalent constellations is proposed. We first take all the samples of original signal higher than the PAPR threshold as the clipping noise, and then minimize the mean squared error of this noise and possible equivalent constellations to determine the optimal size and position of the constellations. In addition, by applying the proposed TI scheme to two special constellation extension, the inherent power increase in TI scheme can effectively be reduced or avoided. Secondly, according to the nonlinear distortion analysis in the presence of power amplifier [27]-[30], a root-mean-square peak interference-to-carrier ratio (RMS-PICR) reduction scheme is also introduced to improve the BER performance. Simulation results confirm that both the proposed PAPR and RMS-PICR reduction schemes can dramatically reduce the computational complexity while maintaining a good system performance.

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The outline of the paper is organized as follows. Section II describes the OFDM system and reviews the tone injection technique for PAPR reduction. Section III presents two tone injection constellation expansion methods that used in this paper. In Section IV, a computationally efficient PAPR reduction TI scheme based on clipping noise is proposed. In Section V, an RMS-PICR reduction TI scheme is introduced to reduce the nonlinear noise in the presence of power amplifier. Section VI analyzes the complexity of the proposed PAPR and RMS-PICR reduction TI schemes. Simulation results are evaluated in Section VII and conclusion in Section VIII.

II. TONE INJECTION FOR PAPR REDUCTION

A. OFDM Systems and PAPR Definition

In a typical OFDM system, data symbols modulated by phase shift keying (PSK) or quadrature amplitude modulation (QAM) are transmitted independently on the subcarriers. Let vector \( \mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]^T \) denote the input data block, where \( N \) is the number of subcarriers in the OFDM system. Thus, a complex baseband representation of an OFDM signal consisting of \( N \) subcarriers is given by

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j 2 \pi k \Delta f t}, \quad 0 \leq t < NT,
\]

where \( j = \sqrt{-1} \), \( \Delta f \) is the subcarrier spacing, and \( NT \) denotes the data block period. The PAPR of the transmitted signal is defined as

\[
PAPR = \frac{\max_{0 \leq t < NT} |x(t)|^2}{1/NT \cdot \int_{0}^{NT} |x(t)|^2 dt}.
\]

In a practical OFDM system, we oversample \( x(t) \) to obtain a discrete-time signal. An \( L \)-times oversampled OFDM signal samples can be achieved by inserting \((L-1) \cdot N\) zeros in the middle of the modulated symbol vector to form the frequency domain data vector \( \mathbf{X} \). The resulting input symbol is expressed as

\[
\mathbf{X} = [X_0, \cdots, X_{N/2-1}, \underbrace{0, \cdots, 0}_{(L-1)N \text{ zeros}}, X_{N/2}, \cdots, X_{N-1}]^T.
\]

Therefore, the \( L \)-times oversampled OFDM signal can be written as

\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j 2\pi nk/LN}, \quad n=0, 1, \cdots, LN - 1.
\]

The PAPR computed from the \( L \)-times oversampled time domain signal samples can be formulated as

\[
PAPR = \frac{\max_{0 \leq n < LN-1} |x_n|^2}{E[|x_n|^2]},
\]

where \( E[\cdot] \) denotes the expectation operation. Following [32], for \( L \geq 4 \), the model in (5) is accurate to approximate continuous-time PAPR.

B. Tone Injection

Tone injection [14] extends the original constellation to several equivalent points so that the same information can be carried by any of these points. For an \( M \)-ary square QAM, the real and imaginary parts of \( X_k \) can take values from the set \( \{ \pm d/2, \pm 3d/2, \cdots, \pm (\sqrt{M} - 1)d/2 \} \), where \( \sqrt{M} \) and \( d \) represent the number of levels per dimension and the minimum distance between constellation points, respectively.

Mathematically, the objective of tone injection is to send the symbols [14]

\[
\tilde{x}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (X_k + p_k \cdot \Delta + j q_k \cdot \Delta) \exp \left( \frac{j 2\pi kn}{LN} \right),
\]

where \( p_k, q_k \) are integers and \( (p_k + j q_k) \cdot \Delta \) can be seen as the extra freedom constellation. In order not to increase BER at the receiver, the value of \( \Delta \) should be at least \( d/\sqrt{M} \) [17]. An example of 16-QAM constellation with eight replications of a given symbol is illustrated in Fig. 1. Since each symbol can be mapped into one of nine equivalent constellation points, these extra freedom degrees can be exploited to reduce the PAPR. However, finding the optimal constellation to obtain the lowest PAPR for \( \tilde{x}_n \) is a nondetermined-polynomial hard problem [17]. Therefore, suboptimal solutions are required.

III. CONSTELLATION OF TONE INJECTION

In order to make a tradeoff between the complexity of the TI scheme and the average power increase, the following two extend constellations are adopted in this paper.

A. Extended Constellation

To prevent a greater power increase, only constellations located on the outer ring could be shifted, and the corresponding equivalent constellations are nearly symmetrical about the
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where \( \Delta_k \) we neglect the phase of \( \alpha \) that \( S \) the original QAM constellation \( X_k \) and the corresponding equivalent point, which can be expressed as [19], [22]

\[
S(X_k) = \begin{cases} 
-\frac{d}{2}p_k - j\frac{d}{2}M'' \quad (p_k > -M', q_k = M') \\
-\frac{d}{2}p_k + j\frac{d}{2}M'' \quad (p_k < -M', q_k = -M') \\
-\frac{d}{2}M'' - j\frac{d}{2}q_k \quad (p_k = M', q_k < M') \\
\frac{d}{2}M'' - j\frac{d}{2}q_k \quad (p_k = -M', q_k > M') \\
\end{cases} \quad \text{if } X_k \in S_{EC} \\
\text{otherwise}
\]

where \( M' = \sqrt{M} - 1 \) and \( M'' = \sqrt{M} + 1 \).

In order to facilitate the analysis and design the proposed algorithm, considering that the extended constellation is nearly symmetric, the transmit signal in Fig. 2 can be rewritten as

\[
\hat{x}_{n_{EC}} = x_{n_{EC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_k + \Delta_k) e^{j2\pi nk/LN},
\]

where \( x_{n_{EC}} \) is the same as \( x_n \), \( \alpha_k \) and \( |\Delta_k| \) \(^1\) are defined as

\[
\begin{cases} 
\alpha_k = 2, & |\Delta_k| = d\sqrt{M}, \quad k \in S_{EC} \\
\alpha_k = 0, & |\Delta_k| = 0, \quad \text{otherwise}
\end{cases}
\]

Note that only \( k \in S_{EC} \) contributes to the peak canceling signal.

### B. Hexagonal Constellation

Hexagonal lattice [34] is the most dense packing of regularly spaced points in two dimensions. In addition, the average power is very important in the design of the hexagonal constellation. Therefore, in order to reduce the PAPR without increasing the signal power, a hexagonal constellation (HC) is introduced to TI [18]. In this case, the area of the decision region for each constellation point is \( \sqrt{3}d^2/2 \), and the new transmit signal is given by

\[
\hat{x}_{n_{HC}} = x_{n_{HC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_k + \Delta_k) e^{j2\pi nk/LN},
\]

where \( \alpha_k \) is defined in (10).

Fig. 3 illustrates an example of the 91-points hexagonal constellation (91-HC). The points marked from ‘1’ to ‘64’ are utilized to take six information bits. Furthermore, note that there are two representations for the symbols ‘38’ to ‘64’ in the outer ring, which can be used to reduce the PAPR. Thus, the 91-HC has the same throughput and minimum distance as the square 64-QAM. Since the ratio of the decision region between square 64-QAM constellation and the 91-HC can be calculated as \( d^2/(\sqrt{3}d^2/2) = 2/\sqrt{3} \), the BER of the 91-HC is slightly worse than that of the square 64-QAM. In this case, the average power of the 91-HC points is 10.36d^2 [18], while that of the square 64-QAM is 10.50d^2. Therefore, there is no average power increase by using 91-HC instead of square 64-QAM. In addition, if we reduce the number of ‘2 representations’ constellation points, the average power of 91-HC can be further reduced.

According to Figs. 2 and 3, since only one equivalent constellation to choose to be shifted or not, the PAPR problem...
of EC and HC TI can be written as the following integer optimization problem

$$\begin{align*}
\text{min } f(\alpha) &= |\hat{x}_n(\alpha)|^2 \\
\text{subject to: } &\alpha \in \{0, 2\}^N.
\end{align*}$$

(12)

The optimization problem (12) has been proved to be a nondetermined-polynomial hard problem [17]. Suboptimal solutions, such as cross-entropy TI [21] and parallel tabu search TI [22], are thus employed. However, both of them apply iterative heuristic searching method to solve the problem (12), and they still need hundreds of FFTs to achieve a considerable PAPR. Therefore, a computationally efficient TI algorithm that based on clipping noise will be proposed in the next section.

IV. TONE INJECTION SCHEME FOR PAPR REDUCTION

In this section, we first determine \( S \), the index set of TI subcarriers which contribute to the peak canceling signal, and then introduce the proposed TI algorithm for PAPR reduction.

A. Index Set of EC-TI Subcarriers for PAPR Reduction

Size of \( S_{EC} \): According to (9), we have

$$|\hat{x}_{n_{EC}}| = \left| x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_k + \Delta_k) e^{j2\pi(nk/LN)} \right| \geq |x_n| - \frac{1}{\sqrt{N}}, \sum_{k \in S_{EC}} |2X_k + \Delta_k|. \quad (13)$$

In order to satisfy the PAPR restriction (\( \hat{x}_n \) smaller than the PAPR threshold), a necessary condition is that \( S_{EC} \) must satisfy

$$\frac{1}{\sqrt{N}} \sum_{k \in S_{EC}} (2|X_k| + |\Delta_k|) \geq |x_n|_{\text{max}} - A, \quad (14)$$

for further simplification, \( X_k \) is replaced by its mean value. Let \( \bar{\epsilon} = E\{|X_k|\} \), then for square M-ary QAM [25]

$$\bar{\epsilon}_{EC} = \frac{4}{M} \sqrt{\frac{6}{M-1}} \sum_{p=1}^{\sqrt{M}/2} \sum_{q=1}^{\sqrt{M}/2} \sqrt{(p-0.5)^2 + (q-0.5)^2}. \quad (15)$$

Therefore, the minimum size \( S_{EC} \) of the EC-TI scheme that satisfies (14) can be calculated as

$$N_{S_{EC}} = \left\lceil \sqrt{N} \left( |x_n|_{\text{max}} - A \right) \frac{2\bar{\epsilon}_{EC} + |\Delta_k|}{\bar{\epsilon}_{EC}} \right\rceil. \quad (16)$$

where [\( x \)] represents the smallest integer greater than \( x \). □

Position of \( S_{EC} \): Clipping method limits the peak envelope of the input signal to a predetermined threshold \( A \), where \( A \) is determined by the saturation level of the power amplifier. Thus, the clipping noise can be calculated as

$$f_n = \begin{cases} x_n - A e^{j\phi} & |x_n| > A \\ 0 & |x_n| \leq A \end{cases}, \quad (17)$$

where \( \phi \) represents the phase of \( x_n \). In this paper, we take the entire samples of peaks higher than \( A \) as the clipping noise [11]

$$\hat{f}_n = \begin{cases} x_n & |x_n| > A \\ 0 & |x_n| \leq A \end{cases}. \quad (18)$$

The difference between (17) and (18) is that the former only contains the part of samples larger than \( A \), while the latter contains the whole samples that exceed \( A \). Although both (17) and (18) can work with the proposed algorithms, note that (18) needs less computations than (17) (Compared to (18), (17) requires additional phase angle computation, two real multiplications and subtraction per clipped sample.). Therefore, (18) is used to generate the clipping noise.

In order to obtain the peak canceling signal, we can project the frequency domain clipping noise \( \hat{f}_k \) to \( X_k \).

$$\hat{f}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P_k X_k e^{j2\pi(nk/LN)}, \quad (19)$$

where

$$P_k = \frac{\Re\{\hat{f}_k X_k^*\}}{|X_k|^2}, \quad (20)$$

\( \Re\{x\} \) represents the real part of \( x \), and \( (\cdot)^* \) is the complex conjugate operation. To further minimize the transmit signal, \( \hat{f}_n \) can be scaled by a factor \( \beta \),

$$\hat{x}_n' = x_n - \beta \hat{f}_n = x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \beta \cdot P_k X_k e^{j2\pi(nk/LN)}, \quad (21)$$

Here, we choose the suboptimal solution [16] by minimizing the out-of-range power to obtain the optimal \( \beta \),

$$\min_{\beta} \sum_{|\hat{x}_n'|>A} (|\hat{x}_n'| - A)^2. \quad (22)$$

If \( \beta \) is a real number, the optimal solution can be calculated as

$$\beta = \frac{\Re\{\sum f_n \hat{x}_n'\}}{\sum |f_n|^2}. \quad (23)$$

The mean rounding error of (9) and (21) is upper bounded as

$$\begin{align*}
\varepsilon_{EC} &= E\{|\hat{x}_{n_{EC}} - \hat{x}_{n_{EC}}'|\} \\
&= \frac{1}{LN} \sum_{n} \left| \frac{1}{\sqrt{N}} \sum_{k \in S_{EC}} (|2 - \beta P_k| X_k + \Delta_k) e^{j2\pi(nk/LN)} \right| \\
&- \frac{1}{\sqrt{N}} \sum_{k \notin S_{EC}} \beta \cdot P_k X_k e^{j2\pi(nk/LN)} \\
&\leq \frac{1}{\sqrt{N}} \left\lceil \sum_{k \in S_{EC}} |(2 - \beta P_k) X_k| + |\Delta_k| \right\rceil + \sum_{k \notin S_{EC}} |\beta P_k X_k| \right\rceil. \quad (24)
\end{align*}$$

Generally, we give the following theorem by minimizing the mean rounding error \( \varepsilon_{EC} \) to achieve the optimal position of \( S_{EC} \).

**Theorem 1**: Define

$$Z_m = |(2 - \beta P_m) X_m| + |\beta P_m X_m|. \quad (25)$$

Therefore, \( \varepsilon_{max} \) is minimized if

$$Z_m < Z_n, \quad \text{for all } m \in S \text{ and } n \notin S. \quad (26)$$
The proof of Theorem 1 is provided in Appendix. Considering the size of \( S \) in (16) and this theorem, it follows that the index set \( S_{EC} \) is made up of the smallest \( N_{S_{EC}} \) of \( Z_m \).

### B. Index Set of HC-TI Subcarriers for PAPR reduction

**Size of \( S_{HC} \):** The difference between extended and hexagonal constellation is that the former is nearly symmetric, while the latter is perfectly symmetric. With the similar analysis of EC-TI, according to (11) and (13), we have

\[
|x_{n_{HC}}| = \left| x_{n_{HC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \alpha_k X_{k_{HC}} e^{j2\pi(nk/LN)} \right| \\
\geq |x_{n_{HC}}| - \frac{1}{\sqrt{N}} \sum_{k \in S_{HC}} |2X_{k_{HC}}|,
\]

and the necessary condition satisfies the PAPR restriction can be calculated as

\[
\frac{2}{\sqrt{N}} \sum_{k \in S_{HC}} |X_{k_{HC}}| \geq |x_{n_{HC}}|_{\text{max}} - A.
\]

Finally, the minimum size of \( S_{HC} \) in HC-TI is given by

\[
N_{S_{HC}} = \left\lceil \sqrt{N} \left( |x_{n_{HC}}|_{\text{max}} - A \right) / 2 \epsilon_{HC} \right\rceil,
\]

where \( \epsilon_{HC} = E\{|X_{k_{HC}}|\} \).

**Position of \( S \):** Similar to Theorem 1, the position of \( S \) in HC-TI is also made up of the smallest \( N_{S_{HC}} \) of \( Z_m \). However, the mean rounding error of (11) and (21) is given by

\[
\epsilon_{HC} = E\{|\hat{x}_{n_{HC}} - \hat{x'}_{n}\| = \frac{1}{LN} \sum_{n} \left( \frac{1}{\sqrt{N}} \sum_{k \in S_{HC}} (2 - \beta P_k) \cdot X_{k_{HC}} e^{j2\pi(nk/LN)} \right) \right. \\
- \left. \frac{1}{\sqrt{N}} \sum_{k \notin S_{HC}} \beta \cdot P_k X_{k_{HC}} e^{j2\pi(nk/LN)} \right| \\
\leq \frac{1}{\sqrt{N}} \left( \sum_{k \in S_{HC}} \left| (2 - \beta P_k) X_{k_{HC}} \right| + \sum_{k \notin S_{HC}} |\beta P_k X_{k_{HC}}| \right).
\]

### C. Proposed PAPR Reduction Tone Injection Schemes

According to Theorem 1, it can easily determine the size and position of the optimal equivalent constellations. Therefore, the proposed algorithm for the EC-TI scheme can now be summarized in Algorithm 1.

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**Algorithm 1: Clipping Noise-based Extended Constellation Tone Injection for PAPR Reduction**

1: Initialization: set up the PAPR threshold \( \xi \) and the maximum iteration number \( T \). Let \( \zeta = |x_n|_{\text{max}} \) and \( \hat{x}_{n_{opt}} = x_n \).

2: for \( t = 1 : T \) do

3: Calculate the signal \( x_n \) and \( PAPR_{\hat{x}_{n_{opt}}} \) according to (4) and (5).

4: if \( (PAPR_{\hat{x}_{n_{opt}}} > \xi) \) then

5: Set the index set of tone injection subcarriers \( S = \emptyset \).

6: Obtain the clipping noise \( f_n \) from (18).

7: Calculate \( N_{S_{EC}}, P_k, \beta, \) and \( Z_m \) by using (16), (20), (23), and (25), respectively.

8: Find the smallest \( N_{S_{EC}} \) of \( Z_m \) to make up the tone injection index set \( S_{EC} \).

9: Let the \( \alpha_i = 2 \) for \( i \in S_{EC} \), and others \( \alpha_i = 0 \).

10: Update the peak-reduced signal \( \hat{x}_{n_{EC}} \) by using (9).

11: if \( (|\hat{x}_{n_{EC}}|_{\text{max}} < \zeta) \) then

12: Store \( \hat{x}_{n_{EC}} \) as the best solution \( \hat{x}_{n_{opt}} \) and let \( \zeta = |\hat{x}_{n_{EC}}|_{\text{max}} \).

13: end if

14: if \( (t = T \) or \( PAPR_{\hat{x}_{n_{opt}}} \leq \xi) \) then

15: Transmit the modified signal \( \hat{x}_{n_{opt}} \).

16: end if

17: else

18: Transmit the original signal \( x_n \).

19: end if

20: end for

---

The proposed HC-TI algorithm is very similar to the EC-TI algorithm, expect the following steps:

- ...

7: Calculate \( N_{S_{HC}} \) by using (29).

- ...

10: Update the peak-reduced signal \( \hat{x}_{n_{HC}} \) by using (11).

- ...

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**V. TONE INJECTION SCHEME FOR RMS-PICR REDUCTION**

### A. Power Amplifier Model

Nonlinear distortions are primarily caused by the transmitter high power amplifier (HPA). Generally, the modeling of nonlinear power amplifiers is very complex [35]. A common simplification is to assume the nonlinearity is memoryless, e.g., for a PA with input signal \( x(t) = |x(t)| e^{j\phi(t)} \), the output signal \( g[x(t)] \) is given by

\[
g[x(t)] = F[|x(t)|] e^{j(\phi(t)+\Phi[x(t)])},
\]

where \( F[x(t)] \) and \( \Phi[x(t)] \) are the amplitude modulation (AM)/AM and AM/phase modulation (PM) conversions of the PA, respectively.

An often used solid state power amplifier (SSPA) model, Rapp model, can be expressed as [36]

\[
F[x(t)] = \frac{x(t)}{1 + \left(\frac{|x(t)|}{C_e}\right)^{2p}}, \Phi[x(t)] = 0,
\]
where $C$ is the saturating amplitude of the PA and the AM/PM conversion is assumed to be negligibly small. Note that the parameter $p$ controls the AM/AM sharpness of the SSPA.

According to [27], the output of a $K$-th order baseband equivalent polynomial model for a nonlinear HPA can be written as

$$y_{out} = \sum_{k=1}^{K} \nu_k y_{in} |y_{in}|^{(k-1)}, \quad \text{(33)}$$

where $K$ and $\nu_k$ are the order of nonlinearity and the polynomial non-linear coefficients, respectively. Consider that the odd order non-linearity causes maximum in-band distortion and $K = 3$ is a good approximation of the intermodulation (IM) power [27], [28], therefore, the PA output can be approximated as

$$y_{out} = \nu_1 \cdot y_{in} + \nu_3 \cdot |y_{in}|^2 y_{in}. \quad \text{(34)}$$

Similar to [30], we can use this polynomial model to modeling the SSPA, that is, solving using curve-fitting to obtain the coefficients. For example, for a SSPA with $p = 3$ and $C = 6$ dB, the parameter $\nu_1$ and $\nu_3$ are 1 and -0.14, respectively.

B. Proposed RMS-PICR Reduction TI Scheme in the Presence of PA

In this section, we will introduce the proposed TI scheme for RMS-PICR reduction in the presence of PA. Combining the definition of FFT and (34), the frequency domain SSPA output signal can be written as

$$R_k = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\nu_1 + \nu_3 \cdot |x_n|^2) \cdot x_n e^{-j2\pi nk/LN}. \quad \text{(35)}$$

In [29], it shows that $R_k$ can be rewritten as

$$R_k = \mu X_k + \psi_k, \quad \text{(36)}$$

where $\mu X_k$ represents the useful attenuated symbol input replica, and $\mu$ is given by

$$\mu = \nu_1 + \frac{\nu_3}{N} \sum_{n=0}^{N-1} |x_n|^2. \quad \text{(37)}$$

The nonlinear noise component $\psi_k$ can be calculated as

$$\psi_k = \frac{\nu_3}{N} \sum_{p=0}^{N-1} \sum_{n=0}^{N-1} X_p |x_n|^{2} e^{j2\pi(p-k)/LN}. \quad \text{(38)}$$

The peak interference-to-carrier ratio (PICR) is introduced in [37]. In this paper, we take the root-mean-square (RMS) PICR as the distortion metric, e.g.,

$$\text{PICR}_{rms} = \left[ \frac{1}{N} \sum_{k=0}^{N-1} \text{PICR}_k \right]. \quad \text{(39)}$$

where $\text{PICR}_k$ is defined as

$$\text{PICR}_k = \left| \frac{\psi_k}{\mu X_k} \right|^2. \quad \text{(40)}$$

In order to minimize the RMS-PICR, we need to minimize the nonlinear noise component $\psi_k$, which is determined by the index set of TI subcarriers $S$. Taking HC-TI as an example, according to (11), we have

$$\left| \tilde{x}_{n_{HC}} \right| \geq \left| x_{n_{HC}} \right| - \left| \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \alpha_n X_{k_{HC}} e^{j2\pi nk/LN} \right| \geq |x_{n_{HC}}| - \frac{2}{\sqrt{N}} \sum_{k \in S_{HC}} |X_{k_{HC}}|. \quad \text{(41)}$$

The $R_k$ of HC-TI can be written as

$$R_{k_{HC}} = \mu (X_{k_{HC}} - \alpha_k X_{k_{HC}}) + \psi_k = \mu (1 - \alpha_k) X_{k_{HC}} + \psi_k, \quad \text{(42)}$$

and its time-domain signal can be calculated as

$$r_{n_{HC}} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mu (1 - \alpha_k) X_{k_{HC}} e^{j2\pi nk/LN} + \Psi_n \quad \text{(38)}$$

where $\Psi_n$ is the time-domain signal of $\psi_k$. Thus,

$$\sum_{k \in S_{HC}} |X_{k_{HC}}| \geq \frac{\sqrt{N}}{2\mu} |\mu x_{n_{HC}} - r_{n_{HC}} + \Psi_n| \geq \frac{\sqrt{N}}{2\mu} [\mu |x_{n_{HC}}| - |r_{n_{HC}}| - |\Psi_n|] \geq \frac{\sqrt{N}}{2\mu} \left( (\mu - \nu_1 - \nu_3 \cdot |x_{n_{HC}}|^2) \cdot |x_{n_{HC}}| - |\Psi_k| \right).$$

Finally, a necessary condition that satisfies (44) in HC-TI is given by

$$N_{S_{HC}}' = \left[ \frac{\sqrt{N}}{2\mu \cdot E[|X_{k_{HC}}|^2]} \left( (\mu - \nu_1 - \nu_3 \cdot |x_{n_{HC}}|^2) \cdot |x_{n_{HC}}| - |\Psi_k| \right) \right]. \quad \text{(45)}$$

Therefore, the proposed RMS-PICR reduction TI scheme is summarized in Algorithm 2.

VI. ANALYSIS OF THE COMPUTATIONAL COMPLEXITY

To find the optimal constellation points in (6), conventional TI requires solving an integer programming problem, which has exponential complexity. Assuming there are $L$ candidates per constellation, if $K$ dimensions are to be shifted, we must search over all $[17]$

$$C_N^K \cdot L^K \approx \frac{N^K}{K!} \approx (NL)^K \quad \text{(46)}$$

combinations, and each combination requires an IFFT operation.

The overall complexity of the proposed EC-TI PAPR reduction scheme is mainly determined by (9) and (19), which need three FFTs per iteration. The complexity of (9) depends on the size of $S_{EC}$. The average size of $S_{EC}$ can be calculated as

$$\bar{N}_{S_{EC}} = \int_{A}^{\infty} N_{S_{EC}}(r) \cdot p(r) \, dr. \quad \text{(47)}$$
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Algorithm 2: RMS-PICR Reduction Tone Injection Scheme

1: Initialization: set up the RMS-PICR threshold $\xi$ and the maximum iteration number $T$. Let $x_n^\text{opt}(t) = x_n$.
2: for $t = 1 : T$ do
3:     Calculate the frequency domain SSPA output signal $R_k$ and the noise component $\psi_k$ according to (35) and (36).
4:     Obtain the RMS-PICR value from (40).
5:     if (RMS-PICR $x_n^\text{opt} > \xi$) then
6:         Set the index set of TI subcarriers $S = \emptyset$.
7:     Calculate $N'_S$ according to (45).
8:     Find the smallest $N'_S$ of $\psi_k$ to make up the tone injection index set $S$.
9:     Let the injection index set.
10: Update the RMS-PICR-reduced signal $\tilde{x}_n$ by using (11).
11: if ($t = T$ or RMS-PICR $x_n^\text{opt} \leq \xi$) then
12:     Transmit the modified signal $x_n^\text{opt}$.
13:     end if
14: else
15:     Transmit the original signal $x_n$.
16:     end if
17: end for

where $N_{SE_C}(r)$ is the size of $S_{EC}$ by replacing $r$ with $|x_n|_{\text{max}}$

$$N_{SE_C} = \left\lfloor \frac{\sqrt{N} (r - A)}{2 \cdot \epsilon_{EC} + |\Delta_k|} \right\rfloor, \quad (48)$$

An approximate analytical cumulative distribution function expression of PAPR has been developed in [38]

$$\Pr(|x_n|_{\text{max}} < r) \approx \exp \left(-Ne^{-r\sqrt{\frac{\pi}{3} \log N}}\right), \quad (49)$$

and the probability density function of PAPR can be found as

$$p(r) = N\sqrt{\frac{\pi}{3} \log N} \exp \left(-r - Ne^{-r\sqrt{\frac{\pi}{3} \log N}}\right). \quad (50)$$

Therefore, for the EC-TI PAPR reduction, the average size of $S$ can be calculated as

$$\bar{N}_{SE_C} = \int_A^\infty \left\lfloor \frac{\sqrt{N} (r - A)}{2 \cdot \epsilon_{EC} + |\Delta_k|} \right\rfloor \cdot p(r) dr, \quad (51)$$

and for the HC-TI PAPR reduction, the average $N_S$ is given by

$$\bar{N}_{SHC} = \int_A^\infty \left\lfloor \frac{\sqrt{N} (r - A)}{2 \cdot \epsilon_{HC}} \right\rfloor \cdot p(r) dr. \quad (52)$$

For example, when $N = 256$, $A = 4$ dB, and 16-QAM, $\bar{N}_{SE_C} \approx 10.2$ and $\bar{N}_{SHC} \approx 11.8$, respectively. Therefore, the FFT complexity of the (9) and (11) is much lower than a full $LN$-length FFT.

Assume $N_f$ is the number of nonzero samples in $f_n$. In [16], it is shown that the mean of $N_f$ is a function of $N$ and can be found as

$$\bar{N}_f = \frac{LNe^{-A^2/2\sigma^2}}{2}. \quad (53)$$

For example, when $A/\sqrt{2\sigma} = 6$ dB, $\bar{N}_f = 1.87 \times 10^{-2}LN$. Since $\bar{N}_f$ is sparse, the complexity of the (19) may be further reduced to $O(N)$ by using a proper wavelet transform [39]. Therefore, the complexity of the (19) is also much lower than that of a $LN$-length FFT. In order to simplify the complexity comparison, we lose the upper bound complexity of the proposed EC and HC TI PAPR reduction schemes as three FFTs per iteration.

With similar analysis, it can be shown that the upper bound complexity of the proposed EC and HC RMS-PICR reduction schemes is two FFTs per iteration, since the RMS-PICR reduction scheme does not need to evaluate the parameter $\beta$ in (19). In the next Section, simulations will show that the proposed TI schemes dramatically reduce the search time to obtain a considerable PAPR and BER performance.

VII. SIMULATION RESULTS

In this section, the complementary cumulative distribution function (CCDF) is used to evaluate the PAPR performance, which is given by

$$CCDF_x(PAPR_0) = \Pr(PAPR > \xi). \quad (54)$$

This is the probability that the PAPR of a symbol exceeds the threshold level $A$. The Monte Carlo simulations below are performed for the random QAM modulated OFDM symbols, with the subcarriers $N$ under the condition of four times oversampling. Furthermore, we choose four PAPR reduction schemes, PTS and SLM [10], cross entropy (CE)-TI [21], parallel tabu search (TS)-TI [22], as well as the original OFDM, for comparison. Our simulations consider an OFDM system with 6 dB SSPA saturating amplitude, 256 subcarriers, the oversampling factor $L = 4$, and the iteration time are shown in the figure description.

Fig. 4 depicts the CCDF curves of various PAPR reduction schemes with the subcarriers $N = 256$. To achieve the same PAPR performance, the complexity of the proposed EC-TI and HC-TI schemes is much lower than the other schemes.
needs eighteen FFTs, while other schemes need hundreds of FFTs. Some comparisons of these algorithms are listed in Table I.

Fig. 5 illustrates the PAPR and computational complexity comparison of the various schemes with 256 subcarriers. A comparison of six PAPR reduction schemes. It can be observed that, with the same complexity, the proposed two TI schemes achieve much smaller PAPR than other schemes. Specially, to achieve a 9 dB PAPR at \(10^{-4}\) clipping probability, the proposed scheme only needs three FFTs, while the CE-TI needs fifty FFTs. For 256 subcarriers at \(10^{-4}\) clipping probability, the PAPR reduction of the proposed HC-TI scheme with eight iterations is about 1.9 dB better than SLM with 24 candidates, 2.1 dB better than PTS with eight random subblock partition. In this case, both the HC-TI and SLM schemes require 24 FFTs per OFDM block, while PTS needs 24 FFTs. Some comparisons of these algorithms are listed in Table I.

Table I

<table>
<thead>
<tr>
<th>Scheme</th>
<th>CE-TI</th>
<th>SLM</th>
<th>PTS</th>
<th>EC-TI</th>
<th>HC-TI</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAPR</td>
<td>8.25 dB</td>
<td>8.07 dB</td>
<td>8.24 dB</td>
<td>6.56 dB</td>
<td>6.15 dB</td>
</tr>
<tr>
<td>Power Increase</td>
<td>1.31 dB</td>
<td>No</td>
<td>No</td>
<td>0.49 dB</td>
<td>No</td>
</tr>
<tr>
<td>Average Runtime</td>
<td>198.4 ms</td>
<td>18.3 ms</td>
<td>75.6 ms</td>
<td>11.6 ms</td>
<td>11.3 ms</td>
</tr>
<tr>
<td>Number of FFTs</td>
<td>200</td>
<td>24</td>
<td>128</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Side Information</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Fig. 6. The power spectrum density of the SLM, PTS, EC-TI and HC-TI schemes with 64QAM and 256 subcarriers.

Specially, to achieve a 9 dB PAPR at \(10^{-4}\) clipping probability, the proposed scheme only needs three FFTs, while the CE-TI needs fifty FFTs. For 256 subcarriers at \(10^{-4}\) clipping probability, the PAPR reduction of the proposed HC-TI scheme with eight iterations is about 1.9 dB better than SLM with 24 candidates, 2.1 dB better than PTS with eight random subblock partition. In this case, both the HC-TI and SLM schemes require 24 FFTs per OFDM block, while PTS needs 24 FFTs. Some comparisons of these algorithms are listed in Table I.

Fig. 5 illustrates the PAPR and computational complexity comparison of the various schemes with 256 subcarriers. A comparison of six PAPR reduction schemes. It can be observed that, with the same complexity, the proposed two TI schemes achieve much smaller PAPR than other schemes. Specially, to achieve a 6.5 dB PAPR, the proposed HC-TI scheme only needs eighteen FFTs, while other schemes need hundreds of FFTs.

Therefore, the power spectrum density (PSD) of some schemes is compared in Fig. 6 under a SSPA. In our simulations, we choose \(p = 3\). As shown in Fig. 6, the proposed HC-TI scheme has much less impact on the power spectrum compared to other schemes. The HC-TI with eight iterations leads to only -40 dB out of band radiation, which is 2 dB lower than EC-TI, 6 dB lower than SLM with 24 candidates, 7 dB lower than PTS with eight subgroups, and 12 dB lower than the original OFDM signal, respectively.

Fig. 7 compares the BER performance of the HC-TI, EC-TI, SLM and PTS under a 64QAM and 256 subcarrier OFDM system. Note that the ‘Original’ curve is obtained by directly transmitting the original OFDM signals through the SSPA. Assume the side information of SLM and PTS is perfect transmitted to the receiver. To achieve the same BER performance (e.g., \(10^{-3}\) BER), the required SNR values under the EC-TI, HC-TI, SLM and PTS with SSPA are 19.2 dB, 19.6 dB,
The cubic metric (CM) [40] is an effective predictor of the actual reduction in power capability of a typical power amplifier. Due to its accuracy over a wide range of devices and signals, it has been adopted by the 3GPP members as a method to determine PA power de-rating. This method has proved to be superior compared to methods that use the statistical PAPR to predict de-rating in OFDM system [41]. The CM of a OFDM signal is defined as

$$CM = \frac{RCM - RCM_{ref}}{K},$$

where $RCM$ is the raw CM. The $RCM$ of a signal $x(t)$ can be written as

$$RCM = 20 \log_{10} \left[ \left( \frac{|x(t)|}{\text{rms}[x(t)]} \right)^3 \right],$$

where the $RCM_{ref}$ and $K$ are used to complete estimate of the power de-rating required to meet a given adjacent channel leakage ratio [41]. For example, in downlink of LTE $RCM_{ref} = 1.52$ dB and $K = 1.56$ are used. Fig. 9 shows this comparison. The proposed HC-TI scheme also achieves the best CM performance among other schemes when the CCDF of CM is evaluated.

**VIII. CONCLUSION**

This paper first presented a novel clipping noise based TI scheme to reduce computational complexity and improve PAPR performance of OFDM signals. By projecting the clipping noise to the nearest equivalent constellations, the proposed scheme easily determines the optimal equivalent constellations. Thus, the number of FFTs needed is substantially reduced. Secondly, a PICR reduction TI scheme is also introduced to further improve the BER performance. Simulation results show that the proposed algorithm achieves a larger PAPR or PICR reduction and lower computational complexity than other schemes.

**APPENDIX**

**PROOF OF THEOREM 1**

Suppose $S$ satisfies (26). Let $\mathcal{N} = \{0, 1, \ldots, N - 1\}$ is the subcarriers index set of OFDM and

$$\mathcal{N} = \bigcup_{i=1}^{4} \mathcal{S}_i,$$

where $\mathcal{S}_i \cap \mathcal{S}_k = \emptyset$, for $i \neq k$. Given that

$$S = \mathcal{S}_1 \cup \mathcal{S}_2, S' = \mathcal{S}_1 \cup \mathcal{S}_3,$$

where the set $S$ and $S'$ have the same size. Fig. 10 shows the relationship of $S$, $S'$, $S_1$, and $\mathcal{N}$. Here, note that $S_2$ and $S_3$ are randomly selected. In the following, we show that $S$ has a smaller maximum mean rounding error (i.e., $\varepsilon$ in (24)) than $S'$. The maximum mean rounding error of (24) caused by $S$ in EC-TI scheme is given by

$$\varepsilon_{\text{max}} = \sum_{k \in \mathcal{S}_1 \cup \mathcal{S}_2} |(2 - \beta P_k)X_k| + |\Delta_k| + \sum_{k \in \mathcal{S}_1 \cup \mathcal{S}_3} |\beta P_k X_k|,
Similarly, based on (30) for HC-TI scheme,

\[
\bar{\varepsilon}_{\text{HC max}} - \bar{\varepsilon}'_{\text{HC max}} = \sum_{k \in S_2} |[(2 - \beta P_k) X_k]| + \sum_{k \in S_3} |\beta P_k X_k| - \sum_{k \in S_2} |[(2 - \beta P_k) X_k]| - \sum_{k \in S_3} |\beta P_k X_k| \\
= \sum_{k \in S_2} Z_k - \sum_{k \in S_3} Z_k \leq 0.
\]

Since \( S_2 \) and \( S_3 \) are randomly selected, and \( S_2 \in S, S_3 \in S' \), \( S \) minimizes the maximum mean rounding error \( \bar{\varepsilon}_{\text{max}} \).

### References


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